UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2020

IS51026A/IS51026B
Numerical Maths

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

The use of calculators is allowed. Students are required to note the model of the calculator on the answer sheet.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Graph paper will be provided.

**THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM**

Part A

Multiple choice

Question 1 Each question has one correct answer

(a) What is the decimal representation of 221_8?

i. 49_{10}
ii. 1160_{10}
iii. 345_{10}
iv. 145_{10}

[2]

(b) What is the binary representation of 221_{16}?

i. 11011101_2
ii. 1000010001_2
iii. 10010001_2
iv. 1000100001_2

[2]

(c) The sequence $1, 2, 4, 8, ...$ is

i. arithmetic
ii. neither arithmetic and geometric
iii. geometric
iv. both arithmetic and geometric

[2]

(d) A sequence is determined by a recurrence relation $u_{n+1} = u_n + 2$, the third term u_3 is $= 6$ find u_1

i. -2
ii. 0
iii. 2
iv. 4

[2]

(e) Given $13 \times 10^6 \equiv 15(mod 17)$ the remainder on division by 17 of $(13 \times 10^6)^2$ is:

i. 4
ii. 225
iii. 16×10^{12}
iv. 169×10^{12}

[2]

(f) The graph of $f(x) = (x-1)(x-2)(x^2+1)$:

i. passes through the point $(0, -2)$
ii. passes through the point $(1, 0)$
iii. has a x-intercept of 1
iv. has a y-intercept of -2

[2]

(g) $\frac{2\pi}{3}$ radians is:

i. 0.0366 degrees
ii. 2.09 degrees
iii. 60 degrees
iv. 120 degrees

[2]

(h) A triangle ABC has sides $a = 17.2$ cm, $b = 16.5$ cm and angle $A = 1.3$ radians. The size of angle B is:

i. 1.18 radians
ii. 1.00 radians
iii. 0.27 radians
iv. This triangle does not exist

[2]

(i) The period of $f(x) = 2\sin(\pi + 2x)$ is

i. 2π
ii. π
iii. $\frac{1}{2\pi}$
iv. $\frac{1}{\pi}$

[2]

(j) The frequency of $f(x) = 2\sin(\pi + 2x)$ is

i. $\frac{1}{2\pi}$
ii. 2π
iii. $\frac{1}{\pi}$
iv. π

[2]

(k) $\log_{10} 1$ is equal to

i. 0
ii. 0.1
iii. $\log_{100} 10$
iv. is not defined

[2]

(l) $3 \log_{10} 10000$ is equal to:

i. $3 \log_2 16$
ii. 300
iii. 3000
iv. is not defined

[2]

(m) Calculate the following limit: $\lim_{x\to\infty} \frac{x^2-x}{x^2+x}$.

i. -1
ii. 1
iii. ∞
iv. is not defined

[2]

(n) Given $y = \cos(3x + \pi)$

i. $\frac{dy}{dx} = \cos(3x + \pi)$
ii. $\frac{dy}{dx} = 3\cos(3x + \pi)$
iii. $\frac{dy}{dx} = -\sin(3x + \pi)$
iv. $\frac{dy}{dx} = -3\sin(3x + \pi)$

[2]

(o) The modulus of the cartesian vector $(-3, 3)$ is

i. $-\sqrt{2}\sqrt{3}$
ii. $\sqrt{-3}\sqrt{3}$
iii. $2\sqrt{3}$
iv. $3\sqrt{2}$

[2]

(p) You are given vectors $\underline{u}=\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ and $\underline{v}=\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$\underline{u} \cdot \underline{v}$ the dot product (scalar product) of $\underline{u}$ and $\underline{v}$ is equal to

i. 2

ii. 5

iii. $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

iv. $\begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$

[2]

(q) Find M^{-1}, the inverse of M where $M = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

i. $\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

ii. $\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

iii. $\begin{pmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$

iv. $\begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix}$

[2]

(r) The following matrix represents which of the following transformations? $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. A scaling

ii. A translation

iii. A rotation

iv. A reflection

[2]

(s) Let S be the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ how many 4 digit codes can be made from set S if repetitions are allowed?

i. 100

ii. 40

iii. 10^4

iv. 10^{10}

[2]

(t) Let S be the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. how many 4 digit codes can be made from set S if repetitions are not allowed, and all codes are $<$3000

i. 2160

ii. 3024

iii. 1512

iv. 63

[2]

Part B

Question 2 Bases, Modular Arithmetic & Probability

(a) i. Express the decimal number $(121.625)_{10}$ as a binary number [2]

ii. Express the octal number $(147.34)_8$ as a decimal number [2]

iii. Express the hexadecimal number $(FE.16)_{16}$ as

(1) a binary number

(2) an octal number [4]

iv. Working in base 2 and showing all your working, compute the following:

$$(1001010)_2 + (1001011)_2 - (10001111)_2$$

[2]

(b) i. Find the smallest positive integer modulo 29 that is congruent to

(1) 184

(2)1540

(3)-1540 [3]

ii. Find the remainder on division by 29 of

(1) $184 + 1540$

(2) 184×1540

(3) 1540^{20} [5]

iii. Find the following

(1) the additive inverse of 184 modulo 29

(2) the multiplicative inverse of 184 modulo 29 [2]

(c) i. How many different numbers can be made:

(1) using the digits 1, 2, 3, 4 once each

(2) the digits 1, 2, 3, 4, 5, 6 once each [4]

ii. How many different numbers can be made :

(1) using the digits 1, 1, 2, 2 once each

(2) using the digits 1, 1, 1, 2, 2, 2 once each [3]

iii. How many different arrangements can be made from:

(1) 4 different coloured beads in a circle, where direction does not matter

(2) 4 beads, 2 black, 2 white in a circle, where direction does not matter [3]

Question 3 Functions, Vectors & Matrices

(a) i. Find numerical values for the following

(1) $\log_3 81$

(2) $\log_3(\frac{1}{81})$

(3) $\log_{\frac{1}{3}}(\frac{1}{81})$

(4) $4\log_{\frac{1}{3}}(\frac{1}{3})$ [3]

ii. Sketch the graphs of

(1) $f(x) = 2^{x+1}$

(2) $g(x) = \log_2(x-1)$ [4]

iii. Find the inverse functions

(1) $f^{-1}(x)$

(2) $g^{-1}(x)$ [3]

(b) Given $\underline{v}_1 = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$

i. Rewrite $\underline{v}_1$ in terms of standard unit vectors

ii. Find the magnitudes of $\underline{v}_1$ and $\underline{v}_2$

iii. Find the dot product (scalar product) $\underline{v}_1 \cdot \underline{v}_2$

iv. State the angle between $\underline{v}_1$ and $\underline{v}_2$

v. Find a vector $\underline{v}_4$ that is parallel to $\underline{v}_1$

vi. Find a vector $\underline{v}_5$ that is perpendicular to $\underline{v}_1$

[10]

(c) Let A be a 3x3 homogeneous transformation matrix corresponding to an anti-clockwise rotation of $\frac{\pi}{2}$ about the z-axis.

Let B be a 3x3 homogeneous transformation matrix corresponding to a translation of the x and y-coordinates by -1 and 1 respectively.

i. Find the matrices A and B [2]

ii. How would the transformation represented by the matrix B transform the following three points which represent a triangle in the Cartesian space: (1,0), (2,0) and (1,1)? [3]

iii. Find the inverse matrices A^{-1} and B^{-1} [2]

iv. Find the single matrix $C = A^2$ [2]

v. Describe the transformation represented by the matrix C [1]

Question 4 Graph sketching, Differentiation, &Trigonometric functions

(a) i. Find the following limits

(1)$\lim_{x\to 1} \frac{1}{x+1}$
(2)$\lim_{x\to 1} \frac{x^2-1}{x-1}$
(3)$\lim_{x\to 1^+} \frac{1}{1-x}$
(4)$\lim_{x\to \infty} \frac{3x^5+2x^2+2}{x^5+1}$
(5)$\lim_{x\to 0} \frac{x^5+1}{3x^5+2x^2+2}$ [5]

ii. Differentiate the following functions:

(1)$y = 3\cos x + 2\sin x$
(2)$y = \frac{\cos x}{\sin x}$
(3)$y = e^{\frac{\cos x}{\sin x}}$, you may use the result for (2) above. [5]

(b) Given the following function $f(x) = (x-1)^2(x+1)^2$

i. Find the values of x for which $f(x) = 0$

ii. Differentiate $f(x)$, (note $((x-1)^2(x+1)^2 = x^4 - 2x^2 + 1)$

iii. Hence find the stationary points of $f(x)$

iv. Determine the nature of each of the stationary points of $f(x)$

v. Sketch $f(x)$ [10]

(c) i. Triangle ABC has side $a = 43.0cm$, side $b = 27.0cm$ and angle $C = 1.80$ radians. Find the following: (give your answers to 3 significant figures)

(1) the length of side c
(2) the size of angles A and B [4]

ii. Given $f(x) = 3\sin x$ and $g(x) = \cos\frac{x}{2}$

(1) Find the amplitude, the frequency and the period for

- $f(x)$
- $g(x)$

[3]

(2) By plotting the graphs of $f(x)$ and $g(x)$, or otherwise, find all the values of x between $-\pi \le x \le \pi$ for which $3\sin x = \cos\frac{x}{2}$ (note $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$) [3]