# UNIVERSITY OF LONDON <br> GOLDSMITHS COLLEGE <br> <br> Department of Computing <br> <br> Department of Computing <br> B. Sc. Examination 2020 <br> <br> IS50003B, IS50003C <br> <br> IS50003B, IS50003C <br> Foundations of Problem Solving <br> Duration: 2 hours 15 minutes <br> <br> Date and time: 

 <br> <br> Date and time:}

This paper is in two parts: part $A$ and part $B$. You should answer $A L L$ questions from part $A$ and TWO questions from part B. Part A carries 40 marks, and each question from part $B$ carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

The use of calculators is allowed. Students are required to note the model of the calculator on the answer sheet.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

## THIS PAPER MUST NOT BE REMOVED <br> FROM THE EXAMINATION ROOM

## Part A

## Question 1

In order to write a program for finding the sum of the multiples of 5 from 1 to 80 (consider both 1 and 80 in your program), you have been asked to perform the following tasks:
i. Write the pseudo code of your algorithm to solve the problem.
ii. Draw the flowchart of your algorithm.

Show clearly the variables used in your algorithm as well as the start and the end of your flowchart.

## Question 2

The list of seven numbers shown below is to be sorted in ascending order.

| 20 | 19 | 15 | 28 | 12 | 16 | 26 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(a)
i. Use bubble sort to perform the first pass, giving the state of the list after each exchange. State the number of comparisons and swaps needed to perform the first pass. Clearly indicate number sorted at the end of the first pass.
ii. Continue with the bubble sort by conducting further passes, showing the state of the list after each pass, until the algorithm terminates. State the total number of comparisons, swaps and how many passes are needed before the algorithm terminates. Clearly indicate numbers sorted at the end of each pass.
(b) Another list of numbers, in descending order, is given below:

$$
\begin{array}{llllllllll}
71 & 65 & 64 & 46 & 45 & 43 & 24 & 23 & 16 & 13
\end{array}
$$

The numbers in the list represent the lengths, in cm, of some pieces of wood required to build a small cabinet. Planks in one metre length can be purchased for $£ 4$ each.
i. Use the first fit decreasing algorithm to determine how many of these Planks are to be purchased to make this cabinet. Find the total cost and the amount of wood wasted.
ii. Determine whether your solution to part b (i) is optimal. Give a reason for your answer.

## Part B

## Question 3 Transportation Problem

The table below shows the cost, in pounds, of transporting one unit stock from each of three supply points, A, B and C to three demand points, X, Y and Z. It also shows the stock held at each supply point and the stock required at each demand point.

|  | X | Y | Z | Supply |
| :---: | :---: | :---: | :---: | :---: |
| A | 20 | 11 | 10 | 25 |
| B | 19 | 15 | 18 | 20 |
| C | 9 | 13 | 12 | 18 |
| Demand | 19 | 18 | 26 |  |

(a) Is this a balanced problem? Justify your answer.
(b) Use the north west corner method to obtain an initial solution and state the cost of the initial solution.
(c) Amongst the negative improvement indices, take the smallest to indicate the entering cell and use the stepping-stone method to obtain an improved solution. You must make your method clear by stating your shadow costs, improvement indices, routes, entering cell and exiting cell.
(d) Perform one more iteration of the stepping-stone method to find a further improved solution. You must state clearly your shadow costs, improvement indices, entering cell, exiting cell and clear route.
(e) State the cost of the solution you found in part (d).

## Question 4 Simplex Algorithm

a. Given that $k$ is a constant, display the following linear programming problem in a Simplex tableau by introducing slack variables to convert inequalities to equalities.

You must write your equations before displaying them in the simplex tableau.

Maximize $\quad P=3 x+5 y+6 z$
Subject to $\quad k x+2 y+z \leq 8$

$$
\begin{aligned}
& x+10 y+2 z \leq 17 \\
& x \geq 0, y \geq 0, z \geq 0
\end{aligned}
$$

b. Use the Simplex Algorithm to perform one iteration of your tableau for part a. You must clearly show any workings in your table that allows you to choose your pivot row. You must also indicate the pivot row, pivot value and state the row operations you use.
c. Given that the optimal value of $P$ has not been achieved after this first iteration, find the range of possible values of $k$.
d. In the case where $k=-1$, perform one further iteration and interpret your final tableau. You must state the final value of each variable (including slack variables) and of the objective function.

## Question 5 Travelling Salesman Problem

The table below shows the least costs, in pounds, of travelling between 6 cities labelled $A, B$, C, D, E and F.

|  | A | B | C | D | E | F |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | - | 36 | 18 | 28 | 24 | 22 |
| B | 36 | - | 54 | 22 | 20 | 27 |
| C | 18 | 54 | - | 42 | 27 | 24 |
| D | 28 | 22 | 42 | - | 20 | 30 |
| E | 24 | 20 | 27 | 20 | - | 13 |
| F | 22 | 27 | 24 | 30 | 13 | - |

Andrea must visit each city at least once. He will start and finish at A and wishes to minimise the total cost.
a) Use Prim's algorithm, starting at A, to find the minimum spanning tree for this network.
b) Using your answer to part a), calculate an initial upper bound for the length of Andrea's route.
c) Show that there are two nearest neighbour routes that start at $A$. You must clearly state your routes and their lengths.
d) State the best upper bound from your answers to part b) and c)
e) By deleting A and all of the arcs connected to it, find a lower bound for the length of the route.
f) Using your results in d) and e), write down the smallest interval which you are confident contains the optimal route length.

END OF EXAMINAMTION

