

UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2020

IS50002A, IS50002B

Foundations of Problem Solving

Duration: 3 hours

Date and time:

You should answer ALL questions on this paper. There are TEN questions in total. Full marks will be awarded for complete answers to all ten questions. Each question is worth 10 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

The use of calculators is allowed. Students are required to note the model of the calculator on the answer sheet.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Graph paper will be provided. A formulae sheet is included.

**THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM**

Formulae sheet

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of the first n terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Sum of the first n terms of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Question 1 Percentages, Fractions, Order of Operations, Rounding, Standard forms.

a) Find

i. $42\frac{1}{2}\%$ as a decimal

ii. $42\frac{1}{2}\%$ as a fraction in simplest form

iii. $42\frac{1}{2}\%$ of 600

[3]

b) Round the following numbers

i. 0.0041993 to 3 significant figures

ii. 23.4598 to 3 decimal places

[2]

c) Subtract 4.2×10^{-1} from 2.4×10^2 and give your answer in standard form. You must show your working

[2]

d) Insert brackets to make the following equation true

$$6 - 4^2 + 2 \times 5 = 30$$

[1]

e) A number is 1550 correct to 3 significant figures. Find

i. The smallest possible number

ii. The largest possible number

[2]

Question 2 Algebraic expressions, Substitution and Ratio

a) Expand and simplify the following

i. $(x - y) - 8(x + y)$

ii. $(x - y)^2 - 8(x + y)$

iii. Evaluate the expression $(x - y)^3(x - 2y)$ when $x = 2$ and $y = -1$

[3]

b) Simplify the following expressions, giving answers in their simplest form

i. $\frac{(4x+5)(x+3)}{(x+3)(x+5)}$

ii. $\frac{x^2+5x+6}{x^2-4x-12}$

[3]

c) Simplify the following expressions

i. $\frac{4x}{3} \div \frac{2}{9}$

ii. $(a^2b^5) \times (a^{-3}b^{-2}c)$

[2]

d) Two brothers, Mike and Vince, share a sum of money in the ratio 3: 8. Vince gets £40 more than Mike. Calculate how much the brothers share. You must show your working.

[2]

Question 3 Number bases, Factors and Multiples

a) Convert

i. $456B_{16}$ to decimal

ii. 653_{10} to base 7

[2]

b) Convert $40F6_{16}$ to

i. Binary

ii. Octal

You must show working.

[2]

c) Find the largest decimal number that can be written with 5 hexadecimal digits

[1]

d) Calculate the following and show full working.

i. $1011_2 + 10011_2$. Both your working and answer must be in binary.

ii. $11001_8 - 1010_8$. Both your working and answer must be in octal.

[3]

e) Express the numbers 120 and 150 as a product of primes and find their lowest common multiple.

[2]

Question 4 Logic and Sets

a) Construct and complete a truth table for the propositions P , Q and R and the following logical expressions

i. $P \rightarrow Q$

ii. $P \wedge (Q \rightarrow R)$

iii. $(P \rightarrow Q) \vee (P \wedge (Q \rightarrow R))$

iv. State whether $(P \rightarrow Q) \vee (P \wedge (Q \rightarrow R))$ is a tautology, a contradiction or a contingency.

[4]

b) Suppose A , B and C are subsets of a universal set \mathcal{E} as follows:

$$\mathcal{E} = \{x: x \text{ is an integer and } -3 \leq x \leq 15\}$$

$$A = \{x: x \text{ is a multiple of } 3\}$$

$$B = \{x: x > 5\}$$

$$C = \{x: x \text{ is an even number}\}$$

List the following sets

i. $A \cap B$

ii. $A \cup \bar{C}$

iii. $(A \cap B) \cup \bar{C}$

[4]

c) Draw and shade a Venn diagram to represent the following general set

$$\bar{X} \cap (\bar{Y} \cup Z)$$

[2]

Question 5 Linear, Simultaneous and Quadratic Equations

a) Solve the following equations. You must show full working.

i. $4x - 6 = -5x + 2$

ii. $3(2x - 8) = -5(2x + 8)$

[2]

b) Solve the following simultaneous equations

$$4x - 3y = 11$$

$$10x + 2y = -1$$

[2]

c) Factorise the following quadratic expressions.

i. $x^2 - 6x + 9$

ii. $4x^2 + 3x$

iii. $4x^2 - 5x - 6$

[3]

d) Hence or otherwise, solve the following quadratic equations

i. $x^2 - 6x + 9 = 0$

ii. $4x^2 + 3x = 0$

iii. $4x^2 - 5x - 6 = 0$

[3]

Question 6 Sequences and Series

a) Given the sequence 11, 7, 3, -1, ...

- i. Find an expression for the n th term of the sequence
- ii. Find an expression for the sum of the first n terms of the sequence
- iii. Find S_{10} , the sum of the first 10 terms

[3]

b) Given the following sum $\sum_{i=0}^3 (-2)^i (3i - 1)$

- i. Write out the sum in full
- ii. Find the value of the sum

[2]

c) Write the following sum using sigma notation

$$(2 \times 3)^2 + (3 \times 4)^2 + (4 \times 5)^2 + \dots + (20 \times 21)^2$$

[2]

d) The second term of a geometric progression is 24. The sum to infinity of this progression is 150.

- i. Write down two equations in a and r where a is the first term and r is the common ratio.
- ii. Solve your equations to find the possible values of a and r .

[3]

Question 7 Graphs

- a)
- i. Plot the graph of $y = 4x + 1$ for $-5 \leq x \leq 5$ and show where it cuts the axes
 - ii. On the same grid, plot the graph of $x^2 - 4$ for $-5 \leq x \leq 5$
 - iii. From your graph or otherwise, state the coordinates of the points of intersection of the two functions.

[4]

- b) The point A has coordinates $(-1, 6)$ and point B has coordinates $(6,3)$.

- i. Find the coordinates of the midpoint of the line segment AB
- ii. Find the length of the line segment AB
- iii. Find the equation of the perpendicular bisector of AB giving your answer in the form $y = mx + c$.

[4]

- c) Sketch the graph of $y = 4^x + 1$ showing clearly where it cuts the axes and indicating any asymptotes.

[2]

Question 8 Functions and Matrices

a) Given the matrix $A = \begin{pmatrix} 2 & x \\ 1 & 3 \end{pmatrix}$

- i. Find x given that A has determinant 2
- ii. Find x given that A has no inverse

[3]

b) Given the triangle T with vertices $(2,0)$, $(2,2)$ and $(0,2)$

- i. Find the image of T under the transformation represented by the matrix

$$A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

- ii. Describe the transformation represented by the matrix A .
- iii. Find A^{-1} , the inverse of A .

[3]

c) Given the following functions $f(x) = 4x + 1$ and $g(x) = \frac{2}{x+1}$, $x \neq -1$

- i. Evaluate $f(3)$
- ii. Evaluate $g(f(4))$
- iii. Find an expression for $f^{-1}(x)$
- iv. Find an expression for $f^{-1}\left(\frac{2}{x+1}\right)$

[4]

Question 9 Probability

a) In a class of 25 students, 8 are in the school band, 5 are in the choir and 3 are in both the choir and school band. A student is chosen at random from the class.

- i. Draw a Venn diagram to represent the information
- ii. Find the probability that the student is not in the band
- iii. Find the probability that the student is neither in the band nor the choir.

[4]

b) A bag contains 12 balls, 5 blue, 3 green and 4 red. A ball is taken from the bag at random, the colour is recorded, and it is not replaced. A second ball is then taken from the bag and its colour recorded.

- i. Draw a tree diagram to model this event
- ii. Find the probability that both balls are the same colour
- iii. Find the probability that one ball is green, and another ball is blue.

[4]

c) A fair coin is tossed twice.

- i. List all the possible outcomes
- ii. Find the probability of getting two heads

[2]

Question 10 Trigonometry and Indices

- a) Triangle ABC has angle $A = 90^\circ$ and sides $a = 13 \text{ cm}$ and $b = 5 \text{ cm}$
- Find the length of side c .
 - Find angles B and C , give your answers to the nearest degree. [3]
- b) Triangle PQR has angle $Q = 35^\circ$ and $R = 105^\circ$ and side $p = 120 \text{ cm}$. Find the length of side r . Give your answer to three significant figures. [2]
- c) Given that $2^3 = 8$, express 8^{2x+4} in the form 2^y where y is an expression in terms of x . [1]
- d)
- Evaluate $9^2 \div 9^5$
 - Evaluate $125^{-\frac{2}{3}}$
 - Given that $27\sqrt{3} = 3^a$, find the value of a .
 - Simplify $(16x^{12})^{\frac{3}{4}}$ [4]

END OF EXAMINATION