UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2019

IS53057A
Advanced Algorithms and Data Structures

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A

Question 1 Parts (a)–(j) require true or false answers with justifications. You should read each statement carefully and then determine whether the statement is TRUE or FALSE. If the statement is true, explain why it is true or give a supportive example. If it is false, explain what should be the correct answer or provide a counterexample.

(a) In the context of complexity theory, suppose we know there is a polynomial reduction from problem X to problem Y. If there is no polynomial algorithm for problem Y, we can conclude that there is no polynomial algorithm for problem X.
True or False? Justify your answer. [4]

(b) For randomised algorithms, the worst-case running time and expected running time are equal to within constant factors.
True or False? Justify your answer. [4]

(c) Huffman coding is not optimal unless all the probabilities of its alphabet is $\frac{1}{4}$.
True or False? Justify your answer. [4]

(d) In dynamic programming, the input of (current) stage m becomes typically the output to (previous) stage $m-1$.
True or False? Justify your answer. [4]

(e) The Quick Sort Algorithm does not need to employ the dynamic programming approach.
True or False? Justify your answer. [4]

(f) If a greedy choice property satisfies the optimal-substructure property, then a globally optimal solution is locally optimal.
True or False? Justify your answer. [4]

(g) In *8-queen problem*, the symmetry of the queens can be used for finding the alternative solution to a known solution.
True or False? Justify your answer. [4]

(h) Assume D is the deterministic part of an algorithm and R is the random part. The framework of a randomised algorithm can be described as:
"random input → [DDDDDDD] → output distribution" instead of
"deterministic input → [DDDDRDD] → output".
True or False? Justify your answer. [4]

(i) In the context of Quantum Computing, the superposition s for a level 2 quantum system can be measured as a value greater than 0 and smaller than $|a_0|^2$, i.e. $0 < s < |a_0|^2$.
True or False? Justify your answer. [4]

(j) The efficiency of a parallel algorithm for p processors is defined as

$$E(n,p) = \frac{S(p)}{p}$$

True or False? Justify your answer. [4]

Part B

Question 2

(a) Describe an *optimal* dynamic programming algorithm to find the maximum product of a contiguous sequence of n positive numbers in array $A[i]$, where $i = 1, \cdots, n$. For example, given the input sequence $A[i] = (0.1, 17, 1, 5, 0.05, 2, 4, 1, 0.7, 0.02, 12, 0.3)$, the output of your algorithm would be "85" because of the subsequence (17, 1, 5). Show all your working including the approach and algorithm. [20]

(b) Argue that your algorithm in part (a) is indeed optimal. [10]

Question 3

(a) Given a linked list of n elements, all of which are stored in an array $A[i]$, where $i = 1, \cdots, n$ of processors. Propose a *parallel algorithm* to compute for each element its rank in the list. What is the time complexity of your algorithm in $O(.)$ notation? Justify your answer. [15]

(b) Consider the adjacency matrices of two digraphs

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

i. Draw the digraphs A and B (i.e. named after the matrices A and B respectively). [4]

ii. Write the transition matrices Ta for A and Tb for B. [4]

iii. We know that Ta^{12} and Tb^{12} are

```
Ta^{12}=
  0.2146776    0.4279835    0.1426612    0.2146776
  0.2139918    0.4293553    0.1426612    0.2139918
  0.2139918    0.4279835    0.1440329    0.2139918
  0.2146776    0.4279835    0.1426612    0.2146776
```

and

```
Tb^{12}=
  0.1667974    0.3330078    0.1667974    0.3333973
  0.1666018    0.3334961    0.1666018    0.3333004
  0.1666018    0.3334961    0.1666018    0.3333004
  0.1666987    0.3332520    0.1666987    0.3333507
```

Does this mean that the random walk on A converges as quickly as that on B? Justify your answer. [7]

Question 4

(a) Consider a task of packing n objects into a minimum number of bins. To simplify the problem, we assume that everything is of two dimensional, i.e. bins and objects are merely rectangles. The height of the n objects are referred to as $h_1, h_2, \cdots, h_n$. The width of the objects are all the same in size and can just be fit in the rectangular bins. The available bins are all of capacity C. [20]

i. Draw a diagram to show one instance of the problem.
ii. Classify the problem based on your algorithmic knowledge.
iii. How 'easy' is this problem from algorithmic point of view?
iv. What is the so-called "greedy approach"?

(b) Describe the *matrix chain multiplication* problem with the aid of an example involving four matrices $A_{7\times3}, B_{3\times2}, C_{2\times5}, D_{5\times6}$. Explain how this problem can be solved by a greedy approach and why dynamic programming would be helpful for this problem. [10]