**UNIVERSITY OF LONDON**

**GOLDSMITHS COLLEGE**

**Department of Computing**

**B. Sc. Examination 2019**

# IS51032A
# Symbolic Mathematics

**Duration: 2 hours 15 minutes**

**Date and time:**

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*This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.*

*There are 100 marks available on this paper.*

*The use of calculators is allowed. Students are required to note the model of the calculators on the answer sheet.*

*Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.*

**THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM**

# Part A

## Multiple choice

**Question 1** Each question has one or more correct answers

(a) Let $A = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}\}$. Which of the following sets represent A using the inclusion rules? More than one answer may apply.

i. $\{2^{-n} : n \in \mathbb{Z}$ **and** $1 \leq n \leq 8\}$

ii. $\{2^{-n} : n \in \mathbb{Z}$ **and** $0 < n < 9\}$

iii. $\{\frac{1}{2n} : n \in \mathbb{Z}$ **and** $0 \leq n \leq 8\}$

iv. $\{\frac{1}{2n} : n \in \mathbb{Z}$ **and** $0 \leq n < 8\}$

[4]

(b) Let $S = \{a, b, c\}$, which one of the following sets represents $\mathcal{P}(S)$?

i. $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

ii. $\{\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

iii. $\{\{a\}, \{b\}, \{c\}, \{a, b\}, a, b, \{b, c\}\}$

iv. $\{\{a\}, \{b\}, \{c\}\}$

[4]

(c) Let $p$ and $q$ and be two propositions where $p$ means '**Sarah is not Happy**' and $q$ means '**Sarah is reading a book**'. Which one of the following logical expressions is a correct formalisation of the following sentence:

**Sarah is either happy or is reading a book, but not both**

i. $\neg p \rightarrow q$

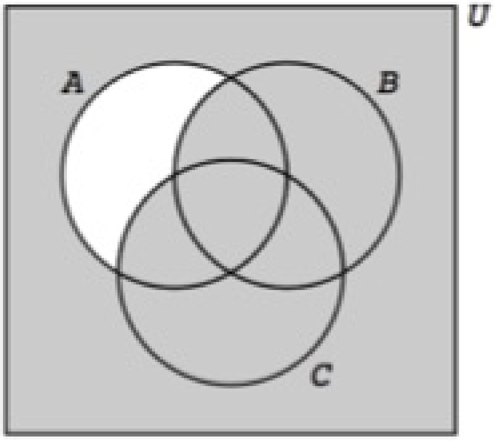
ii. $q \rightarrow \neg p$

iii. $\neg p \oplus q$

iv. $p \oplus \neg q$

[4]

(d) Let $A$, $B$ and $C$ be three subsets of the universal set $U$. Which one of the following sets correctly represents the shaded area in the following Venn diagram

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i. $\bar{A} \vee B \vee C$

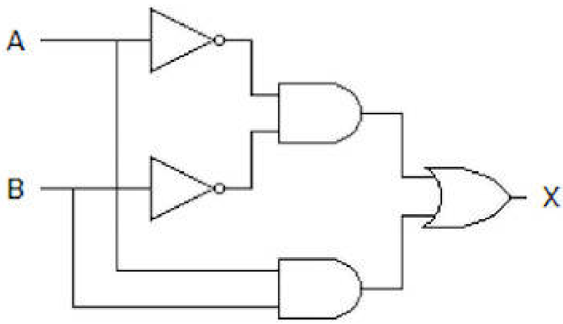
ii. $\bar{A} \vee (B \wedge C)$

iii. $\bar{A} \wedge B \wedge C$

iv. $\bar{A} \oplus (B \vee C)$

‘ [4]

(e) Which one is a correct output of the following logic network:

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i. $(A \wedge B) \vee (\neg A \wedge \neg B)$

ii. $(A \wedge B) \vee (\neg A \wedge B)$

iii. $(A \wedge B) \vee ( A \wedge \vee B)$

iv. $(A \vee B) \wedge (\neg A \vee \neg B)$

[4]

(f) Let $f : R^+ \to R$ be a function where $f(x) = \log_{10} x$. Which one of the following is the inverse function of the function $f$?

i. $f^{-1}(x) = e^x$
ii. $f^{-1}(x) = 10^x$
iii. $f^{-1}(x) = \sqrt[10]{x}$
iv. $f^{-1}(x) = \frac{x}{10}$

[4]

(g) The following sequence $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \cdots$ is

i. arithmetic
ii. geometric
iii. neither geometric nor arithmetic
iv. both arithmetic and geometric

[4]

(h) Let $p$ and $q$ be two propositions. Which one of the following compound statements is equivalent to $p \vee \neg(p \wedge q)$

i. $(p \vee \neg p) \wedge (p \vee \neg q)$
ii. $\neg p \vee \neg q$
iii. $T$
iv. $F$

[4]

(i) Which of the following statements is/are **TRUE**? More than one answer might apply.

i. it is possible to draw a 5-regular graph with 5 vertices
ii. it is possible to draw 2-regular graph with 5 vertices
iii. the sum of the degree sequence of a graph is twice the number of edges in the graph
iv. the sum of the degree sequence of a graph is twice the number of vertices in the graph.

[4]

(j) The number of of edges in a complete graph with n vertices is

i. n-1
ii. n(n-1)
iii. n(n-1)/2
iv. 2n

[4]

# Part B

**Question 2** Set, Logic & Sequences

(a) i. Describe the set $A$ by the listing method.

$$A = \{r^{-2} : r \in \mathbb{Z} \textbf{ and } -1 < r \leq 4\}.$$

ii. Describe the set $B$ using the set builder notation (inclusion rules) where $B = \{1, 3, 9, 15, 81, 243\}$.

[4]

(b) Let $A$ and $B$ and $C$ be subsets of a universal set $\mathcal{U}$.

1. Draw a labelled Venn diagram depicting $A, B, C$ in such a way that they divide $\mathcal{U}$ into 8 disjoint regions. [2]

2. The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

| $A$ | $B$ | $C$ | $X$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Shade the region $X$ on your diagram. Describe the region you have shaded in set notation as simply as you can. [5]

(c) Let $A = \{x : x \in \mathbb{N} \textbf{ and } x < 20\}$. Let $p$ and $q$ be the following propositions concerning a positive integer $n$ in $A$.

$p$ : ' *n is a prime number* '

$q$ : '*n is less than 10* '.

i. Express each of the three following compound propositions concerning positive integers symbolically by using $p$, $q$ and appropriate logical symbols.

*'n is a prime number if it is less than 10'*

*'n is a prime number only if it is less than 10'*

*'n is prime number or a positive integer less than 10, but not both'*

[3]

ii. Find the truth set p for the statement for the logical $q \to p$. [3]

iii. Write in words the contrapositive of the statement given symbolically by '$q \to p$'. [3]

(d) i. Express the following sum using the $\sum$ notation [2]

$$1 + 3 + 5 + 7 + ... + (2n + 1)$$

ii. Evaluate the following the sum: [2]

$$\sum_{k=21}^{100} (2k - 1)$$

Hint: you might want to use the formula: $\sum\limits_{k=1}^{n} k = \frac{n(n+1)}{2}$

iii. Given the following sequence: $u_1 = 1$ and $u_{n+1} = u_n + n + 1$ **for all** $n \geq 1$.

1. Calculate $u_1$, $u_2$. [1]
2. Prove by induction that: $u_n = \frac{n(n+1)}{2}$, **for all** $n \geq 1$. [5]

**Question 3** Graphs & Trees

(a) Name two algorithms used to find the minimum spanning tree of a weighted graph. Give a short explanation of each one of them. [6]

(b) i. Is it possible to construct a 3-regular graph with 5 vertices ? Explain your answer. [2]

ii. Is it possible to construct a simple graph with the degree sequence 5,4,3,2,2? Explain your answer. [2]

iii. Draw the two graphs with adjacency lists. [4]

- $a_1 : a_2, a_5$
- $a_2 : a_1, a_3, a_4, a_5$
- $a_3 : a_2, a_4, a_5$
- $a_4 : a_2, a_3, a_5$
- $a_5 : a_1, a_2, a_3, a_4$

and

- $b_1 : b_2, b_3, b_4, b_5$
- $b_2 : b_1, b_5$
- $b_3 : b_1, b_4, b_5$
- $b_4 : b_1, b_3, b_5$
- $b_5 : b_1, b_2, b_3, b_4$

1. Write down the degree sequence for each graph above. [2]
2. Are these graphs isomorphic? If so, show the correspondence between them. [3]

iv. Define a spanning tree of a graph. [2]

v. How many vertices in a trees with n edges? [1]

vi. A binary search tree is designed to store an ordered list of 3000 records, numbered 1,2,3,...,3000 at its internal nodes.

Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level and find the height of this tree? [8]

**Question 4** Functions, Relations & Probabilty

(a) Let $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2 - 1$

i. List the co-domain and the range of $f$.

ii. Find the ancestors if any of 8.

iii. Is $f$ a one to one function? Explain your answer.

iv. Is $f$ an onto function? Explain your answer.

v. is $f$ an invertible function? Explain your answer.

[9]

(b) Ten balls - five green, three blue, two yellow - are placed in a hat. Two balls are drawn, one after the other, without replacement.

i. What is the probability that both balls are green?

ii. What is the probability that none of the balls are green?

iii. What is the probability that at least one will be green?

[6]

(c) Given $S$ be the set of integers $\{1, 2, 3, 4, 5, 6, 7, 8\}$. Let $\mathcal{R}$ be a relation defined on $S$ by the following condition such that,
for all $x, y \in S$, $xRy$ if $x - y \ mod\ 2 = 0$.

i. Draw the digraph of $\mathcal{R}$. [3]

ii. is $\mathcal{R}$ a partial order? Explain your answer. [2]

iii. is $\mathcal{R}$ a total order? Explain your answer. [2]

iv. Show that $\mathcal{R}$ is an equivalence relation. [6]

v. Write down the equivalence classes of $\mathcal{R}$. [2]