UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2019

IS51026A/IS51026B Numerical Maths

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A

Multiple choice

Question 1 Each question has one correct answer

(a) What is the decimal representation of 152_{16}?

i. 337_{10}
ii. 593_{10}
iii. 145_{10}
iv. 338_{10}

[2]

(b) Which of the following is not a rational number?

i. 2
ii. $\sqrt{8}$
iii. $\sqrt{16}$
iv. 21.212121...

[2]

(c) What is the multiplicative inverse of 8 in modulo 11?

i. 7
ii. 8
iii. 9
iv. 10

[2]

(d) A right angled triangle ABC has angle $A = 0.75$ radians, side $a = 9$ cm and c is the hypotenuse. The size of side b is

i. 0.82 radians
ii. 8.38 cm
iii. 9.66 cm
iv. This triangle does not exist

[2]

(e) A triangle XYZ has sides $x = 7$ cm, $y = 8$ cm and angle $Z = 1.2$ radians. The length of side z is:

i. 12.4 cm
ii. 2.93 cm
iii. 8.51 cm
iv. This triangle does not exist

[2]

(f) Convert 23^o to radians

i. 0.201 radians
ii. 0.401 radians
iii. 0.585 radians
iv. 0.803 radians

[2]

(g) The period of $f(x) = 3\sin(2 + x)$ is

i. 2π
ii. 2
iii. 3π
iv. 3

[2]

(h) The amplitude of $f(x) = 3\sin(2 + x)$ is

i. 2π
ii. 2
iii. 3π
iv. 3

[2]

(i) $3\log_2 8$ is equal to:

i. 24
ii. 9
iii. $\log_2 24$
iv. is not defined

[2]

(j) $\log_{10} -1$ is equal to

i. 0
ii. -1
iii. -0.1
iv. is not defined

[2]

(k) The graph of 2^{x+1}:

i. has a x-intercept of 1
ii. has a y-intercept of 1
iii. passes through the point $(0, 2)$
iv. passes through the point $(2, 0)$

[2]

(l) Calculate the following limit: $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

i. 2
ii. ∞
iii. $\frac{1}{2}$
iv. is not defined

[2]

(m) Given $y = \cos(x^2)$

i. $\frac{dy}{dx} = -\sin 2x$
ii. $\frac{dy}{dx} = \sin 2x$
iii. $\frac{dy}{dx} = -\sin(x^2)$
iv. $\frac{dy}{dx} = -2x \sin(x^2)$

[2]

(n) Given $y = \frac{1}{x}$

i. $\frac{dy}{dx} = \frac{1}{1}$
ii. $\frac{dy}{dx} = -\frac{1}{x^2}$
iii. $\frac{dy}{dx} = \ln x$
iv. $\frac{dy}{dx} = e^x$

[2]

(o) Convert the vector $(2, 2)$ in polar coordinates to cartesian coordinates

i. $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$
ii. $\begin{pmatrix} 2\sqrt{2} \\ \frac{\pi}{2} \end{pmatrix}$
iii. $\begin{pmatrix} 2\sqrt{2} \\ \frac{3\pi}{2} \end{pmatrix}$
iv. none of the above

[2]

(p) You are given vectors $\underline{u}=\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\underline{v}=\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$

$\underline{u} \times \underline{v}$ the cross product (vector product) of $\underline{u}$ and $\underline{v}$is equal to

i. 3

ii. 4

iii. $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

iv. $\begin{pmatrix} 10 \\ -1 \\ 5 \end{pmatrix}$

[2]

(q) Find M^{-1}, the inverse of M where $M = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

i. $\begin{pmatrix} \frac{1}{2} & 0 & -1 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ii. $\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

iii. is undefined

iv. none of the above

[2]

(r) The following matrix represents which of the following transformations? $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. A scaling

ii. A translation

iii. A rotation

iv. A reflection

[2]

(s) Given complex numbers $z_1 = 2 - i$ and $z_2 = 1 + i$ find $z_1 \times z_2$.

i. $1 + 3i$
ii. $2 + i - i^2$
iii. $1 - 3i$
iv. $3 + i$

[2]

(t) Given complex numbers $z_1 = 2$ and $z_2 = 1 + i$ find $\frac{z_1}{z_2}$.

i. $1 - i$
ii. $-1 + i$
iii. $\frac{1+i}{2}$
iv. $\frac{1+i}{4}$

[2]

Part B

Question 2 Bases, Modular Arithmetic & Trigonometry

(a) i. Express the decimal number $(81.375)_{10}$ as a binary number [2]

ii. Express the hexadecimal number $(1F4.E)_{16}$ as a decimal number [2]

iii. Express the octal number $(173.16)_8$ as

(1) a binary number

(2) a hexadecimal number [4]

iv. Working in base 16 and showing all your working, compute the following:

$$(4AA2)_{16} + (394)_{16} - (1F92)_{16}$$

[2]

(b) i. Find the smallest positive integer modulo 13 that is congruent to

(1) 162

(2)1662 [2]

ii. Find the remainder on division by 13 of

(1) $162 + 1662$

(2) 162×1662

(3) 1662^{19} [6]

iii. Find the following

(1) the additive inverse of 11 modulo 13

(2) the multiplicative inverse of 11 modulo 13 [2]

(c) i. Triangle ABC has side $a = 5cm$, side $b = 6.2cm$ and angle $B = 0.873$ radians Find

(1) the size of angle A

(2) the size of angle C

(3) the length of side c [4]

ii. Given $f(x) = \sin(x + \frac{\pi}{4})$ and $g(x) = 2\cos 2x$

(1) Find the amplitude, frequency and period for

- $f(x)$
- $g(x)$

[3]

(2) By plotting the graphs of $f(x)$, or otherwise, find all the values of x between $-\pi$ and π for which $\sin(x + \frac{\pi}{4}) = 0.5$ [3]

Question 3 Functions, Graph Sketching & Vectors

(a) i. Find numerical values for the following

(1) $\log_2 8$

(2) $\log_2(\frac{1}{4})$

(3) $\log_4(\frac{1}{2})$ [3]

ii. Sketch the graphs of

(1) $f(x) = 3^{-x}$

(2) $g(x) = \log_3 x - 1$ [4]

iii. Find the inverse functions

(1) $f^{-1}(x)$

(2) $g^{-1}(x)$ [3]

(b) i. Find the following limits

(1)$\lim_{x\to 2} \frac{x^2+x}{x+1}$

(2) $\lim_{x\to 0} \frac{x^2+x}{x+1}$

(3)$\lim_{x\to \infty} \frac{x^2+x}{x+1}$ [3]

ii. Given the following function $f(x) = (x-1)(x+1)^2$

(1) Find the values of x for which $f(x) = 0$

(2) Differentiate $f(x)$, (note $(x-1)(x+1)^2 = x^3 + x^2 - x - 1$)

(3) Hence find any stationary points of $f(x)$ and determine their nature

(4) Sketch $f(x)$ [7]

(c) Given $\underline{v}_1 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$

i. rewrite $\underline{v}_1$ in terms of standard unit vectors

ii. Find the magnitudes of $\underline{v}_1$ and $\underline{v}_2$

iii. Find the dot product $\underline{v}_1 \cdot \underline{v}_2$

iv. Hence find the angle between $\underline{v}_1$ and $\underline{v}_2$

v. Find $\underline{v}_3$, the cross product (vector product) $\underline{v}_1 \times \underline{v}_2$

vi. State the angle between $\underline{v}_3$ and $\underline{v}_1$

[10]

Question 4 Matrices & Complex Numbers

(a) Let A be a 3x3 homogeneous transformation matrix corresponding to a scaling of the y-coordinates only by a factor of 3. Let B be a 3x3 homogeneous transformation matrix corresponding to a translation of the x and y-coordinates by -1 and 1 respectively. Let C be a 3x3 homogeneous transformation matrix corresponding to an anti-clockwise rotation of π about the z-axis

i. Find the matrices A, B and C [3]

ii. How would the transformation represented by the matrix C transform the following three points which represent a triangle in the Cartesian space: (1,0), (0,1) and (2,1)? [3]

iii. Find the inverse matrices A^{-1}, B^{-1} and C^{-1} [3]

iv. Find the single matrix D which represents the transformation represented by matrix B followed by transformation represented by matrix A [3]

v. Find the inverse matrix D^{-1} [3]

(b) Given complex numbers $z_1 = 2 + i$ and $z_2 = 3 - i$

i. Represent z_1 and z_2 on an Argand diagram [2]

ii. Find

(1) $z_1 + z_2$

(2) $z_1 - z_2$

(3) $z_1 \times z_2$

(4) $\overline{z_2}$

(5) $\frac{z_1}{z_2}$ [5]

iii. Convert z_1

(1) to polar form

(2) to exponential form [3]

iv. Find ${z_1}^3$, give your answer in exponential form [2]

v. Find all roots ${z_1}^{\frac{1}{3}}$ [3]