UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

BSc Examination 2019

IS50002A and IS50002B
Foundation Mathematics for Computing

Duration: 3 hours

Date:

Time:

You should answer all the questions on this paper. There are ten questions in total. Full marks will be awarded for complete answers to all ten questions. Each question is worth 10 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Electronic calculators may be used. The make and model should be specified on the script. The calculator must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Graph paper will be provided. A formulae sheet is included.

THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM

Formulae sheet

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of _n_ terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of _n_ terms of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

Sine rule

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\,cosA$$

Question 1
This question is about percentages, fractions, order of operations, rounding, standard form and factors

a) Find

    i. 24% as a decimal
    ii. 24% as a fraction in simplest form
    iii. 24% of 3000

                [3]

b) Insert brackets to make the following equation true

$$5 - 3^2 \div 2 - 5 = -3$$

                [1]

c) Round the following numbers

    i. 312.0001 to 2 decimal places
    ii. 312.0001 to 2 significant figures

                [2]

d) Write the following numbers in standard form

    i. (145×10^{-1})
    ii. (0.017×10^3)

                [2]

e) Given the numbers 105 and 45, find their

    i. highest common factor
    ii. lowest common multiple

                [2]

Question 2
This question is about algebraic expressions and substitution

a)
  i. Expand and simplify the following expressions

    i) $(x + y) - (x - y)$
    ii) $(x + y)(x - y)$
    iii) $(x + y)^2(x - y)$

[3]
    iv) Evaluate the expression $(x + y)^2(x - y)$ when $x = 1$ and $y = -2$

[1]

b) Simplify the following expressions, give answers in their simplest form

  i. $\dfrac{(x+2)(3x+4)}{(x+2)(x+4)}$

  ii. $\dfrac{x^2-x-6}{x^2+x-2}$

[2]

c) Simplify the following expressions, give answers as a single fraction in its simplest form

  i. $1 - \dfrac{x}{x+1}$
  ii. $1 \div \dfrac{x}{x+1}$

[2]

d) Simplify the following expressions

  i. $(a^2b^2c^{-1})^2$
  ii. $(a^2b^2c^{-1})^2(bc^2)^{-1}$

[2]

Question 3
This question is about number bases, and factors and multiples

a) Convert the following numbers to decimal

   i. 232_5
   ii. 232_{16}

                                                             [2]

b) Convert the decimal number 232_{10} into

   i. base 16
   ii. base 2
   iii. base 5

                                                             [3]

c)
   i. Find the largest decimal number that can be written with 10 binary digits
   ii. Find the largest decimal number that can be written with 5 octal digits
   iii. Find the largest decimal number that can be written with n binary digits

                                                             [3]

d) Calculate the following

   i. $11011_2 + 1101_2$, show your calculation and give your answer in binary
   ii. $11011_2 - 1101_2$, show your calculation and give your answer in binary

                                                             [2]

Question 4
This question is about logic and sets.

a)
  i. Construct and complete truth tables for the following logical expressions

  i) $(P \to Q) \to P$
  ii) $P \to (Q \wedge P)$
  iii) $(P \to Q) \to R$

  [4]
  ii. Hence or otherwise, find the simplest logical expressions that are logically equivalent to

  i) $(P \to Q) \to P$
  ii) $P \to (Q \wedge P)$

  [2]

b)
  i. $A, B,$ and C are subsets of a universal set $\mathcal{E}$ as follows

$\mathcal{E} = \{and, but, for, nor, or, so, yet\}$

  $A = \{the\ words\ with\ 2\ letters\}$
  $B = \{for, nor, or\}$
  $C = \{the\ words\ with\ an\ o\}$

  List the following sets:
  i) $(A \cup \bar{B}) \cap C$
  ii) $A \cup (\bar{B} \cap C)$

  [2]
  ii. Draw and shade a Venn diagram to show the following general set
$$(X \cup \bar{Y}) \cap Z$$

  [2]

Question 5
This question is about linear, simultaneous and quadratic equations

a) Solve the following equations

  i. $3x - 4 = -5x + 20$
  ii. $3(x - 4) = -5(x + 4)$

[2]

b) Solve the following simultaneous equations

$$\begin{cases} 3x - 4y = 10 \\ -5x - 2y = -8 \end{cases}$$

[2]

c)
  i. Factorize the following quadratic expressions

    (1) $3x^2 - 4x$
    (2) $x^2 + 3x - 4$
    (3) $3x^2 - 4x - 4$

[3]

  ii. Hence or otherwise solve the following quadratic equations

    (1) $3x^2 - 4x = 0$
    (2) $x^2 + 3x - 4 = 0$
    (3) $3x^2 - 4x - 4 = 0$

[3]

Question 6   **This question is about sequences and series.**

a) Given the sequence $9, 5, 1 \ldots$

   i. Find an expression for the nth term in the sequence a_n
   ii. Find an expression for S_n the sum of the first n terms of the sequence
   iii. Find the smallest value of n for which S_n is negative

[3]

b) Given the series $1 + \frac{3}{2} + 2 + \frac{5}{2} \ldots + 20$

   i. Find the number of terms in the series
   ii. Find the sum of the series

[3]

c) Given the following sum $\sum_{j=0}^{3}(j + 1)(j + 2)$
   i. Write out the sum in full
   ii. Find the total
   iii. Rewrite the sum with limits $j=1$ to $j=4$
     ($\sum_{j=0}^{3}(j + 1)(j + 2) = \sum_{j=1}^{4} \ldots$)

[4]

Question 7   **This question is about functions and matrices**

a) Given the following functions

$$f(x) = \frac{1}{x+2}, x \neq -2 \qquad g(x) = x + 2 \qquad h(x) = 2x$$

   i.   Evaluate the following

      i) $g(0)$
      ii) $g(h(0))$

[2]

   ii.   Find expressions for

      i) $h(x+2)$
      ii) $g(h(x))$

[2]

   iii. Find the inverse functions

      i) $h^{-1}(x)$
      ii) $f^{-1}(x)$

[3]

b) Given the triangle T with vertices $(1,0), (1,1)$ and $(0,2)$

   i.   Find the image of T under the transformation represented by matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
   ii.   State the transformation represented by the matrix A
   iii. Find the inverse matrix A^{-1}

[3]

Question 8   This question is about trigonometry

Give your answers to the nearest degree or to 3 significant figures

a) Triangle ABC has angle $B = 90°$ and sides $b = 8cm$ and $c = 5cm$

    i. Find the length of side a
    ii. Find angles A and C

                                                      [3]

b) The triangle XYZ has angles $X = 38°$ and $Y = 107°$ and side $z = 160cm$. Find the length of side x

                                                      [2]

c)

    i. Draw the graph of $y = 2\sin x$ for $-360° \leq x \leq 360°$, give the coordinates of the x and y-intercepts

                                                      [3]

    ii. Using your graph, or otherwise, find all the values of x between $-360°$ and $360°$ for which $2\sin x = 1$

                                                      [2]

Question 9   **This question is about graphs**

a)
   i. Plot the graph of $y = \frac{1}{x^2-1}$ for $-5 \le x \le 5$

      Give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

                                                        [3]

   ii. Use your graph to find solutions to 1 decimal place, if any, to the following equation. Show your method clearly on the graph.

$$\frac{1}{x^2 - 1} = 5$$

                                                         [2]

b) Given the line segment with end points $(-2, 4)$ and $(6, -12)$

   i. Find the midpoint of the line segment
   ii. Find the gradient of the line segment
   iii. Find the equation of the line that passes through the 2 points

                                                         [3]

c) Sketch the graph of $y = 2^{1-x}$ for $-4 \le x \le 4$
Give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

                                                         [2]

Question 10   This question is about probability

a) You have two fair spinners. One spinner is numbered 1, 2, 3, 4.
   The other spinner is numbered 0, 1, 2, 3. You spin both spinners
   and find the difference between the numbers. Find the probability
   of getting an outcome:

   i.   Equal to 0
   ii. Greater than 0
   iii. Less than 0

   [3]

b) A weighted coin is tossed twice. Given that the probability of
   getting 2 heads is $\frac{1}{16}$ find the probability of getting:

   i.   A head on the first toss
   ii. Two tails
   iii. A head and a tail

   [3]

c) You have a bag containing 3 black balls and 5 red balls. You pick 2
   balls without replacement. Find the probability that you pick

   i.   Two black balls
   ii. A black ball on the first pick
   iii. Exactly one black ball
   iv. A black ball on the second pick

   [4]

END OF EXAMINATION