Formulae sheet

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Sum of *n* terms of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

Sine rule

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc\,cosA$$

Question 1
This question is about percentages, fractions, order of operations, rounding and standard form

a) Find

    i. 36% as a decimal
    ii. 36% as a fraction in simplest form
    iii. 36% of 2000

$$[3]$$

b) Insert brackets to make the following equation true
$$4 - 3^2 + 2 \times 5 = 15$$

$$[1]$$

c) Round the following numbers

    i. 312.0001 to 2 decimal places
    ii. 312.0001 to 2 significant figures

$$[2]$$

d) Given that a bag of flour weighs 1kg to the nearest 10g find

    i. The largest possible weight of the flour
    ii. The smallest possible weight of the flour

$$[2]$$

e) Calculate $(1.7 \times 10^{-1}) \times (7.1 \times 10^{-1})$, give your answer in standard form

$$[2]$$

Question 2
This question is about algebraic expressions and substitution

a)

   i. Expand and simplify the following expressions

      1) $(x + y) - 8(x - y)$
      2) $(x + y) \times (x - 8y)$
      3) $(x + y)^2 \times (x - 8y)$

                                                            [3]
      4) Evaluate the expression $(x + y) \times (x - 8y)$ when $x = 1$ and $y = 2$

                                                            [1]

b) Simplify the following expressions, give answers in their simplest form

   i. $\dfrac{x+2}{x^2-3x-10}$

   ii. $\dfrac{x^2-3x-10}{(x+2)(x+4)}$

                                                              [2]

c) Simplify the following expressions, give answers as a single fraction in its simplest form

   i. $1 - \dfrac{2}{x+1}$

   ii. $1 \div \dfrac{2}{x+1}$

                                                              [2]

d) Simplify the following expressions

   i. $a^2 b^2 c^{-1}(bc^2)$

   ii. $a^2 b^2 c^{-1}(bc^2)^2$

                                                              [2]

Question 3
This question is about number bases, and factors and multiples

a) Convert the following numbers to decimal

   i. 1212_3
  ii. 1212_{16}

                                                 [2]

b) Convert the decimal number 1212_{10}

   i. to base 8
  ii. to base 2

                                                 [2]

c) Find the largest decimal number that can be written with 4 hexadecimal digits

                                                 [1]

d) Given that $x000_8 = E00_{16}$ find the value of x where x is an octal digit

                                                 [2]

e) Calculate the following

   i. $10101_2 - 1101_2$, show your calculation and give your answer in binary
  ii. $10101_8 - 1101_8$, show your calculation and give your answer in octal

                                                 [3]

Question 4
This question is about logic and sets.

a)
  i. Construct and complete truth tables for the following logical expressions

    1) $(P \rightarrow P) \rightarrow Q$
    2) $P \rightarrow (P \rightarrow Q)$

                                                      [4]

  ii. Find simple logical expressions that are logically equivalent to

    1) $(P \rightarrow P) \rightarrow Q$
    2) $P \rightarrow (P \rightarrow Q)$

                                                      [2]

b)
  i. $A, B,$ and C are subsets of a universal set $\mathcal{E}$ as follows

$\mathcal{E} = \{x : x \text{ is an integer and } 0 < x \leq 10\}$
$A = \{4, 5\}$
$B = \{1, 2, 3, 4, 5, 6\}$
$C = \{4, 5, 6, 9\}$

    List the following sets:
    1) $\overline{A \cup B} \cap C$
    2) $A \cup \overline{B \cap C}$

                                                      [2]

  ii. Draw and shade a Venn diagram to show the following general set

$$\overline{X \cup Y} \cap Z$$

                                                      [2]

Question 5
This question is about linear, simultaneous and quadratic equations

a) Solve the following equations

    i. $2x - 5 = 5x - 2$
    ii. $2(x - 5) = 5(x - 2)$

                                                           [2]

b) Solve the following simultaneous equations

$$\begin{cases} 2x - 5y = 0 \\ 5x - 2y = 21 \end{cases}$$

                                                           [2]

c)
    i. Factorize the following quadratic expressions

        1) $6x^2 - 16x$
        2) $x^2 + 6x - 16$
        3) $6x^2 - 16x + 10$

                                                        [3]
    ii. Hence or otherwise solve the following quadratic equations

        1) $6x^2 - 16x = 0$
        2) $x^2 + 6x - 16 = 0$
        3) $6x^2 - 16x + 10 = 0$

                                                        [3]

Question 6   This question is about sequences and series.

a) Given the sequence $18, 6, 2 \dots$

   i.   Find an expression for the nth term in the sequence a_n
   ii. Find an expression for S_n the sum of the first n terms of the sequence
   iii. Find S_{10} the sum of the first 10 terms of the sequence, give your answer as a fraction or an integer
   iv. Find S_∞ the sum to infinity of the sequence

   [4]

b) Given the series $2 + 6 + 10 + \cdots + 402$

   i.   Find the number of terms in the series
   ii. Find the sum of the series

   [3]

c) Find the value of the following

$$\sum_{i=3}^{4} (-1)^i \times 3i$$

   [1]

d) Write the following using sigma notation
   $2^3 + 3^4 + 4^5 + \cdots + 10^{11}$

   [2]

Question 7   This question is about functions

Given the following functions

$$f(x) = \frac{2}{x}, x \neq 0 \qquad g(x) = x^2 - 2 \qquad h(x) = \frac{x}{2}$$

a) Evaluate the following

   i. $f(2)$
   ii. $h(2)$
   iii. $g(h(2))$

[3]

b) Find expressions for

   i. $h(4x^2 - 6)$
   ii. $g(h(x))$
   iii. $f(h(x))$

[3]

c) Find the inverse functions

   i. $h^{-1}(x)$
   ii. $f^{-1}(x)$

[2]

d) Find the inverse of the function $f(h(x))$
   (you may use your answer to b) iii above)

[2]

Question 8   This question is about trigonometry

Give your answers to the nearest degree or to 3 significant figures

a) Triangle ABC has angle $B = 90°$ and sides $b = 12cm$ and $c = 5cm$
   Find the length of side a

[1]

b) In the triangle XYZ angle $Y = 90°$, angle $X = 78°$ and side $x = 10\ m$

   i.   Find the length of side y
   ii. Find the size of angle Z

[2]

c) The triangle DEF has angles $D = 48°$ and $F = 57°$ and sides $e = 16cm$. Find the length of side d

[2]

d)
   i.   Draw the graph of $y = 2 - \cos x$ for $-180° \leq x \leq 180°$, give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

[3]
   ii. Using your graph, or otherwise, find all the values of x between $-180°$ and $180°$ for which $2 - \cos x = 1.5$

[2]

a)

   i. Plot the graph of $y = \frac{1}{x^2} - 1$ for $-5 \leq x \leq 5$

      Give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

[3]

   ii. Use your graph to find solutions, if any, to the following equation. Show your method clearly on the graph.

$$\frac{1}{x^2} - 1 = 5$$

[2]

b) Given the line segment with end points $(-2, 4)$ and $(6, -12)$

   i. Find the length of the line segment between the two points
   ii. Find the midpoint of the line segment between the two points
   iii. Find the gradient of the line passing through these points
   iv. Find the equation of the line passing through these points
   v. Find the equation of the line that is perpendicular to this line segment that passes through the point $(0, -2)$

[5]

END OF EXAMINATION