

Formulae sheet

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of n terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Sum of n terms of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Question 1

This question is about percentages, fractions, order of operations, rounding and standard form

a) Find

- i. 36% as a decimal
- ii. 36% as a fraction in simplest form
- iii. 36% of 2000

[3]

b) Insert brackets to make the following equation true

$$4 - 3^2 + 2 \times 5 = 15$$

[1]

c) Round the following numbers

- i. 312.0001 to 2 decimal places
- ii. 312.0001 to 2 significant figures

[2]

d) Given that a bag of flour weighs 1kg to the nearest 10g find

- i. The largest possible weight of the flour
- ii. The smallest possible weight of the flour

[2]

e) Calculate $(1.7 \times 10^{-1}) \times (7.1 \times 10^{-1})$, give your answer in standard form

[2]

Question 2

This question is about algebraic expressions and substitution

a)

i. Expand and simplify the following expressions

i) $(x + y) - 8(x - y)$

ii) $(x + y) \times (x - 8y)$

iii) $(x + y)^2 \times (x - 8y)$

[3]

iv) Evaluate the expression $(x + y) \times (x - 8y)$ when $x = 1$ and $y = 2$

[1]

b) Simplify the following expressions, give answers in their simplest form

i. $\frac{x+2}{x^2-3x-10}$

ii. $\frac{x^2-3x-10}{(x+2)(x+4)}$

[2]

c) Simplify the following expressions, give answers as a single fraction in its simplest form

i. $1 - \frac{2}{x+1}$

ii. $1 \div \frac{2}{x+1}$

[2]

d) Simplify the following expressions

i. $a^2b^2c^{-1}(bc^2)$

ii. $a^2b^2c^{-1}(bc^2)^2$

[2]

Question 3

This question is about number bases, and factors and multiples

a) Convert the following numbers to decimal

i. 1212_3

ii. 1212_{16}

[2]

b) Convert the decimal number 1212_{10}

i. to base 8

ii. to base 2

[2]

c) Find the largest decimal number that can be written with 4 hexadecimal digits

[1]

d) Given that $x000_8 = E00_{16}$ find the value of x where x is an octal digit

[2]

e) Calculate the following

i. $10101_2 - 1101_2$, show your calculation and give your answer in binary

ii. $10101_8 - 1101_8$, show your calculation and give your answer in octal

[3]

Question 4

This question is about logic and sets.

a)

- i. Construct and complete truth tables for the following logical expressions

i) $(P \rightarrow P) \rightarrow Q$

ii) $P \rightarrow (P \rightarrow Q)$

[4]

- ii. Find simple logical expressions that are logically equivalent to

i) $(P \rightarrow P) \rightarrow Q$

ii) $P \rightarrow (P \rightarrow Q)$

[2]

b)

- i. $A, B,$ and C are subsets of a universal set \mathcal{E} as follows

$$\mathcal{E} = \{x: x \text{ is an integer and } 0 < x \leq 10\}$$

$$A = \{4, 5\}$$

$$B = \{1, 2, 3, 4, 5, 6\}$$

$$C = \{4, 5, 6, 9\}$$

List the following sets:

i) $\overline{A \cup B} \cap C$

ii) $A \cup \overline{B \cap C}$

[2]

- ii. Draw and shade a Venn diagram to show the following general set

$$\overline{X \cup Y} \cap Z$$

[2]

Question 5

This question is about linear, simultaneous and quadratic equations

a) Solve the following equations

i. $2x - 5 = 5x - 2$

ii. $2(x - 5) = 5(x - 2)$

[2]

b) Solve the following simultaneous equations

$$\begin{cases} 2x - 5y = 0 \\ 5x - 2y = 21 \end{cases}$$

[2]

c)

i. Factorize the following quadratic expressions

i) $6x^2 - 16x$

ii) $x^2 + 6x - 16$

iii) $6x^2 - 16x + 10$

[3]

ii. Hence or otherwise solve the following quadratic equations

i) $6x^2 - 16x = 0$

ii) $x^2 + 6x - 16 = 0$

iii) $6x^2 - 16x + 10 = 0$

[3]

Question 6 **This question is about sequences and series.**

a) Given the sequence 18, 6, 2 ...

- i. Find an expression for the n^{th} term in the sequence a_n
- ii. Find an expression for S_n the sum of the first n terms of the sequence
- iii. Find S_{10} the sum of the first 10 terms of the sequence, give your answer as a fraction or an integer
- iv. Find S_∞ the sum to infinity of the sequence

[4]

b) Given the series $2 + 6 + 10 + \dots + 402$

- i. Find the number of terms in the series
- ii. Find the sum of the series

[3]

c) Find the value of the following

$$\sum_{i=3}^4 (-1)^i \times 3i$$

[1]

d) Write the following using sigma notation

$$2^3 + 3^4 + 4^5 + \dots + 10^{11}$$

[2]

Question 7 **This question is about functions and matrices**

a) Given the following functions

$$f(x) = \frac{2}{x}, x \neq 0 \qquad g(x) = x^2 - 2 \qquad h(x) = \frac{x}{2}$$

i. Evaluate the following

i) $h(2)$

ii) $g(h(2))$

[2]

ii. Find expressions for

i) $h(4x^2 - 6)$

ii) $g(h(x))$

[2]

iii. Find the inverse functions

i) $h^{-1}(x)$

ii) $f^{-1}(x)$

[2]

b) Given the triangle T with vertices $(1, 0)$, $(0, 1)$ and $(1, 2)$

i. Find the image of T under the transformation represented by

matrix $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

ii. State the transformation represented by the matrix A

iii. Find the inverse matrix A^{-1}

iv. State the transformation represented by the matrix A^{-1}

[4]

Question 8 **This question is about trigonometry**

Give your answers to the nearest degree or to 3 significant figures

- a) Triangle ABC has angle $B = 90^\circ$ and sides $b = 12\text{cm}$ and $c = 5\text{cm}$
Find the length of side a

[1]

- b) In the triangle XYZ angle $Y = 90^\circ$, angle $X = 78^\circ$ and side $x = 10\text{ m}$

- i. Find the length of side y
- ii. Find the size of angle Z

[2]

- c) The triangle DEF has angles $D = 48^\circ$ and $F = 57^\circ$ and sides $e = 16\text{cm}$. Find the length of side d

[2]

d)

- i. Draw the graph of $y = 2 - \cos x$ for $-180^\circ \leq x \leq 180^\circ$, give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

[3]

- ii. Using your graph, or otherwise, find all the values of x between -180° and 180° for which $2 - \cos x = 1.5$

[2]

Question 9 **This question is about graphs**

a)

- i. Plot the graph of $y = \frac{1}{x^2} - 1$ for $-5 \leq x \leq 5$

Give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

[3]

- ii. Use your graph to find solutions, if any, to the following equation. Show your method clearly on the graph.

$$\frac{1}{x^2} - 1 = 5$$

[2]

b) Given the line segment with end points $(-2, 4)$ and $(6, -12)$

- i. Find the gradient of the line segment
ii. Find the length of the line segment
iii. Find the midpoint of the line segment

[3]

c) Draw a graph of the function $y = \log_2(x + 1)$ for $-4 \leq x \leq 4$

Give the coordinates of the x and y-intercepts, if any, and mark any asymptotes.

[2]

Question 10 **This question is about probability**

a) You have two fair spinners. One spinner is numbered 1, 2, 3, 4. The other spinner is numbered 0, 1, 2, 3. You spin both spinners and find the difference between the numbers. Find the probability of getting an outcome:

- i. Equal to 0
- ii. Greater than 0
- iii. Less than 0

[3]

b) A weighted coin is tossed twice. Given that the probability of getting 2 heads is $\frac{1}{16}$ find the probability of getting:

- i. A head on the first toss
- ii. Two tails
- iii. A head and a tail

[3]

c) You have a bag containing 3 black balls and 5 red balls. You pick 2 balls without replacement. Find the probability that you pick

- i. Two black balls
- ii. A black ball on the first pick
- iii. Exactly one black ball
- iv. A black ball on the second pick

[4]

END OF EXAMINATION