## UNIVERSITY OF LONDON

## GOLDSMITHS COLLEGE

## Department of Computing

B. Sc. Examination 2017

## IS50003B, IS50003C

Foundations of Problem Solving
Duration: 2 hours 15 minutes
Date and time:

This paper is in two parts: part $A$ and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.
The use of calculator is allowed. Students are required to note the model of the calculator in the answer sheet.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

## THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

## Part A

## Question 1

In order to write a program for finding the answer to $\sum_{n=1}^{100} n-\sum_{n=1}^{10} n$, you have been asked to perform the following tasks:
i. Write the pseudo code of your algorithm to solve the problem.
ii. Draw the flowchart of your algorithm.

Show clearly the variables used in your algorithm as well as the start and the end of your algorithm.

## Question 2

The aim is to sort the numbers shown below in descending order

$$
20,17,25,13,24,16,22,16
$$

(a) i. For the sorting purpose, use bubble sort to perform the first pass, giving the state of the list after each exchange. And state the number of comparisons needed to perform the first pass.
ii. Continue with the bubble sort and perform further passes, showing the state of the list after each pass, until the algorithm terminates. State how many passes are needed in total before the algorithm terminates.

At this stage the numbers are sorted. Each one of the sorted numbers represents the size of item to be placed into bins of size 40 .
(b) i. Show the result of using the first fit decreasing bin packing algorithm to this situation.
ii. State with reason whether your solution to question b.(i) has used the minimum number of bins.

## Part B

## Question 3

Sarah has a small bird food stall at the local market. Every week, she makes and sells three types of packs A, B and C.

Pack A contains 4 kg of bird seed, 2 suet blocks and 1 kg of peanuts.
Pack B contains 5 kg of bird seed, 1 suet block and 2 kg of peanuts.
Pack C contains 10 kg of bird seed, 4 suet blocks and 3 kg of peanuts.
Each week Sarah has 140 kg of bird seed, 60 suet blocks and 60 kg of peanuts available for the packs.

The profit made on each pack of $\mathrm{A}, \mathrm{B}$ and C sold is $3.50,3.50$ and 6.50 respectively. Sarah sells every pack on her stall and wishes to maximise her profit, P pence.

Let $\mathrm{x}, \mathrm{y}$ and z be the numbers of packs $\mathrm{A}, \mathrm{B}$ and C sold each week.
An initial Simplex tableau for the above situation is

| Basic variable | x | y | z | r | s | t | Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| r | 4 | 5 | 10 | 1 | 0 | 0 | 140 |
| s | 2 | 1 | 4 | 0 | 1 | 0 | 60 |
| t | 1 | 2 | 3 | 0 | 0 | 1 | 60 |
| P | -350 | -350 | -650 | 0 | 0 | 0 | 0 |

(a) Explain the meaning of the variables r , s and t in the context of this question.
(b) Perform one full iteration of the Simplex algorithm to create a new tableau T. Take the most negative number in the profit row to indicate the pivotal column.
(c) State the value of every variable as given by tableau T
(d) Write down the profit equation given by tableau T .
(e) Use your profit equation to explain why tableau T is not optimal.
(f) Taking the most negative number in the profit row to indicate the pivotal column, identify clearly the position of the next pivotal element.

## Question 4

The figure below shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.
(b) State the capacities of cuts $C_{1}$ and $C_{2}$.
(c) Looking at the figure below, state the values along DH, FH, FI and IT, which would complete the labelling procedure. You are not required to copy the network in your answer book. Simply answer the following:


D $\rightarrow \mathrm{H}:$
$\mathrm{H} \rightarrow \mathrm{D}:$
$\mathrm{F} \rightarrow \mathrm{H}$ :
$\mathrm{H} \rightarrow \mathrm{F}$ :
$\mathrm{F} \rightarrow \mathrm{I}:$
$\mathrm{I} \rightarrow \mathrm{F}:$
$\mathrm{I} \rightarrow \mathrm{T}$ :
$\mathrm{T} \rightarrow \mathrm{I}:$
(d) Looking at the diagram given, increase the flow by a further 4 units. You must list each flow-augmenting route you use, together with its flow.
(e) Prove that the flow is now maximal.

## Question 5

An alumnus from Goldsmiths, who has a good job, decides to take the Problem Solving students and staff to a theatre. Unfortunately the alumnus could not get enough seats left for all to see the same play.
In total, there are three different plays (play 1,2 and 3 ) in the theatre. Students will need to have student tickets and the members of staff will have staff tickets.

The table below shows the price of the tickets. There are 18 members of staff and 200 students. Also there are 94, 65, and 80 seats available for plays 1, 2 and 3 respectively.
(a) Complete the table below:

|  | Staff | Student | Dummy | Seats available |
| :---: | :---: | :---: | :---: | :---: |
| Play 1 | $£ 5$ | $£ 4.50$ |  |  |
| Play 2 | $£ 4.20$ | $£ 3.80$ |  |  |
| Play 3 | $£ 4.60$ | $£ 4$ |  |  |
| Tickets needed |  |  |  |  |

(b) Give reason why dummy column is needed in the table above.
(c) Use the north-west corner method to obtain a possible solution. Then take the most negative improvement index to indicate the entering square, use stepping stone method once to obtain the improved solution. Your shadow costs and improvement indices must be shown clearly.
After another iteration the table becomes:

|  | Staff | Student | Dummy |
| :--- | :---: | :---: | :---: |
| Play 1 |  | 73 | 21 |
| Play 2 | 18 | 47 |  |
| Play 3 |  | 80 |  |

(d) Clearly demonstrate that this solution gives the minimum cost and state its value.

