

Formulae sheet

The quadratic formula

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Sum of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Question 1 This question is about order of operations, percentages, standard form and significant figures

a) Insert brackets to make the following equations true:

i. $4 - 2^2 + 5 - 2 + 1 = 2$

ii. $4 - 2^2 + 5 - 2 + 1 = 6$

[2]

b) Convert

i. $\frac{7}{8}$ to a percentage

ii. 82% to a fraction in its simplest form

[2]

c) Round the following numbers

i. 0.300355 to 2 decimal places

ii. 0.300355 to 2 significant figures

iii. 70.535 to 2 decimal places

iv. 70.535 to 2 significant figures

[4]

d) Calculate $(4.2 \times 10^{-1}) \div (8.4 \times 10^3)$. Give your answer in standard form.

[2]

Question 2 **This question is about algebraic expressions and substitution**

a) Expand and simplify the following expressions.

i. $8(x + y) - (x - 8y)$

ii. $(x - 2)^2$

iii. $(x + y)(x - 2)^2$

[3]

b) Evaluate the following expression when $x = 1$ and $y = 2$

i. $8(x + y) - (x - 8y)$

[1]

c) Simplify the following expressions, give answers in their simplest form

i. $\frac{x^2+2}{3x^2+6}$

ii. $\frac{(x+2)^2}{3x^2+6x}$

[2]

d) Simplify the following expressions, give answers as a single fraction in its simplest form

i. $\frac{2}{9} - \frac{x+1}{3}$

ii. $\frac{2}{9} \div \frac{x+1}{3}$

[2]

e) Simplify the following expressions.

i. $ab^2(b^2c)$

ii. $ab^2(b^2c)^{-2}$

[2]

Question 3 This question is about number bases, and factors and multiples

a) Convert the following numbers to decimal

i. 10001111_2

ii. 443_5

[2]

b) Convert

i. the decimal number 76_{10} to base 2

ii. the decimal number 276_{10} to base 8

[2]

c) Convert the hexadecimal number 613_{16} to:

i. binary

ii. octal

[2]

d) Calculate the following:

i. $100011_2 - 10101_2$, give your answer in binary

ii. $372_8 + 237_8$, give your answer in octal

[2]

e) Find

i. the highest common factor of 68 and 80

ii. lowest common multiple of 68 and 80

[2]

Question 4 **This question is about logic and sets.**

a)

- i. Construct and complete a truth table for the following logical expression.

$$(P \rightarrow Q) \wedge (P \rightarrow \neg Q) \quad [3]$$

- ii. Find a simpler expression that is logically equivalent to $(P \rightarrow Q) \wedge (P \rightarrow \neg Q)$

[2]

b) $A, B,$ and C are subsets of a universal set \mathcal{E} as follows:

$$\mathcal{E} = \{x: x \text{ is an integer and } 0 < x \leq 10\}$$

$$A \cap B = \{4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B \cap C = \{5, 6\}$$

$$B \cup C = \{4, 5, 6, 9\}$$

- i. Draw and shade a Venn diagram to represent $A, B,$ and $C,$ include all the elements of the universal set \mathcal{E}

[3]

- ii. List the following sets:

i) $\overline{A \cup B}$

ii) $A \cap \overline{B}$

iii) $\overline{A \cup B} \cap C$

[2]

Question 5 **This question is about linear, simultaneous and quadratic equations**

a) Solve the following equations.

i. $2x - \frac{5}{2} = 3 - 9x$

ii. $2x = 3 - 3(x - 9)$

[2]

b) Solve the following simultaneous equations.

$$\begin{cases} 3a - 2b = 17 \\ 4a + 3b = 17 \end{cases}$$

[2]

c) Factorize the following

i. $4x^2 - 1$

ii. $x^2 - 6x - 16$

iii. $2x^2 - 9x - 5$

[3]

d) Hence or otherwise solve the following quadratic equations

i. $4x^2 - 1 = 0$

ii. $x^2 - 6x - 16 = 0$

iii. $2x^2 - 9x - 5 = 0$

[3]

Question 6 **This question is about sequences and series.**

a) Given the sequence 3, 8, 13, 18 ... 108

i. Find an expression for the n^{th} term in the sequence

ii. Find the number of terms in the sequence

iii. Find the sum of the following series

$$3 + 8 + 13 + 18 + \dots + 108$$

[4]

b) Find the sum to infinity of the following series:

$$128 + 64 + 32 + \dots$$

[2]

c) Write out the following sum in full

$$\sum_{i=3}^6 (-3)^{i-2}$$

[2]

d) Write the following using sigma notation

$$1 \times 3 + 2 \times 4 + 3 \times 5 + \dots + 10 \times 12$$

[2]

Question 7 **This question is about functions and matrices**

Consider the following functions:

$$f(x) = \frac{1}{2x+1}, x \neq -\frac{1}{2} \qquad g(x) = -x^2$$

a) Evaluate the following.

i. $g(0)$

ii. $g(f(0))$

[2]

b) Find an expression for:

i. $g(2x)$

ii. $f(g(x))$

[2]

c) Find the inverse function $f^{-1}(x)$

[2]

d) Given the triangle T with vertices $(1, 0)$, $(0, 1)$ and $(2, 2)$

i. Find the image of T under the transformation represented by

$$\text{matrix } A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

ii. State the transformation represented by the matrix A

[3]

e) Given the matrix M

$$M = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

Find the inverse matrix M^{-1}

[2]

Question 8 **This question is about trigonometry**

Give your answers to the nearest degree or to 2 significant figures

a) Triangle ABC is a triangle with sides 12, 16 and 20cm. Determine whether ABC is a right angled triangle or not. Justify your answer [1]

b) The triangle ABC is an isosceles triangle with side $a = 8\text{cm}$ and angle $A = 50^\circ$

- i. Find all 3 possible sizes of angle B
- ii. Find all 3 possible lengths of side b

[3]

c) In the triangle DEF angle $D = 43^\circ$ side $e = 6.8\text{m}$ and $f = 3.5\text{m}$

- i. Find the length of side d
- ii. Find the size of angle E

[2]

d)

- i. Draw the graph of $y = \sin(x + 30)$ for $0^\circ \leq x \leq 360^\circ$, show where it cuts the axes
- ii. Using your graph, or otherwise, find all the values of x between 0° and 360° for which $\sin(x + 30) = 0.3$

[4]

Question 9 **This question is about graphs**

- a) Plot the graph of $y = 3x + 6$ for $-5 \leq x \leq 5$ [1]
- b) On the same grid plot the graph of $y = x^2 + 2$ for $-5 \leq x \leq 5$ [2]
- c) Use your graphs to find solutions to the following simultaneous equations. Show your method clearly on the graph.

$$\begin{cases} y = 3x + 6 \\ y = x^2 + 2 \end{cases}$$

[2]

- d)
- Find the equation of the line that passes through $(-2, -4)$ and $(6, 12)$.
 - State its gradient
 - State the y-intercept
- [3]
- e) Draw a graph of the function $y = 2^{-x}$ for $-5 \leq x \leq 5$
Show clearly where it cuts the axes and mark any asymptotes

[2]

Question 10 This question is about probability

a) You have two fair dice. You roll both dice and find the sum of the numbers. Find the probability of getting an outcome of:

- i) 11
- ii) 5 or 6

[2]

b) A fair coin is tossed 3 times.

- i. Draw a tree diagram to represent this process.
- ii. What is the probability of getting 3 heads
- iii. What is the probability of getting 3 heads or 3 tails

[3]

c) You have a bag containing 3 black balls, 2 red balls and 2 green balls. You pick 2 balls without replacement. Find the probability that:

- i. The first ball is green
- ii. The second ball is green
- iii. Both the first ball and the second ball are green
- iv. Neither the first ball nor the second ball are green
- v. Exactly one of the balls is green

[5]

END OF EXAMINATION