UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2016

IS51026A
Numerical Mathematics

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] *brackets.*

There are 100 marks available on this paper.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A

Multiple choice

Question 1 Each question has one correct answer

(a) The binary number 11111111 in decimal is

i. 2^8
ii. $2^8 - 1$
iii. 2^7
iv. none of the above

[4]

(b) The binary number 10101.1 in decimal is

i. 20.5
ii. 20.25
iii. 21.5
iv. none of the above

[4]

(c) It is correct to say (in base 2) that $0111 > 111$?

i. True
ii. False

[4]

(d) It is correct to say (in base 2) that $1000 > 0111$?

i. True
ii. False

[4]

(e) Which one of the following sets is a subset of $\{2, 4, 6, 8, 10, 12\}$?

i. $\{14\}$
ii. $\{2, 3, 4\}$
iii. $\{4, 8, 12\}$
iv. $\{1, 3, 5\}$

[4]

(f) Let A, B be two subsets of a universal set U. Which of of the following describes $A \oplus B$

i. the set of elements contained in A and in B.
ii. the set of elements contained in A or in B.

iii. the set of elements containted in A or in B but not in both.

iv. the set of elements contained in A but not in both.

[4]

(g) Let A be a set of some elements. Which one of the following is correct:

i. $A \in \mathcal{P}(A)$

ii. $A \subseteq \mathcal{P}(A)$

iii. $\emptyset \subseteq \mathcal{P}(A)$

iv. None of the above

[4]

(h) Which of the following numbers is an irrational number.

i. 2.00005

ii. π

iii. $\frac{1}{2}$

iv. $3.1212\cdots$

[4]

(i) If $f(x) = 3x^2 - 2x - 5$, what is the value of f (-1)?

i. -4

ii. -10

iii. -6

iv. 0

[4]

(j) The value of the angle 235 in radian is

i. $\frac{\pi}{235}$

ii. $\frac{235}{\pi}$

iii. $\frac{235\pi}{180}$

iv. $\frac{180\pi}{255}$

[4]

Part B

Question 2 Number Systems & Sets

(a) i. Working in base 2 and showing all your working, compute the following:

$$(10101)_2 + (11011)_2 - (101)_2$$

ii. Express the hexadecimal number $(D08.1C)_{16}$ in base 2.

iii. Express the decimal number $(347)_{10}$ in base 2.

iv. Express the binary number $(110101001.011)_2$ as

- a decimal number
- a hexadecimal number
- an octal number

[12]

(b) i. Describe the set A by the listing method.

$$A = \{3r - 1 : r \in Z and\ -1 < r \leq 5\}.$$

ii. Describe the set B by the rule of inclusion method where $B = \{2, 4, 8, 16, ...1024\}$

[6]

(c) Let A and B and C be subsets of a universal set $\mathcal{U}$.

i. Draw a labelled Venn diagram depicting A, B, C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions.

ii. The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can.

[12]

Question 3 Functions

(a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d\}$ two sets. Let f be a function defined as follows:

$f : A \to B$

x	1	2	3	4	5	6
$f(x)$	a	b	a	c	d	d

i. Draw the arrow diagram to represent the function f .

ii. List the co-domain and the range of f.

iii. Find the ancestor (pre-image) of d.

iv. Show that f is not a one to one function.

v. Show that f is an onto function.

[10]

(b) Consider the function $f(x) = 2\sin 2x$.

i. What is the period of the function f?

ii. Find the amplitude of the function f?

iii. Fill in the missing values in the following table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2\sin 2x$					

iv. Plot the graph of f for x in $[-\pi, \pi]$.

[10]

(c) Let $f(x) = x^3 - 3x + 2$

i. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

ii. Work out the first and second derivatives of the function f (f' and f").

iii. Find all stationary points of the function f and their nature i.e. maxima, minima or inflection point.

iv. Plot the curve of the function f.

[10]

Question 4 Matrices & Transformations

(a) Given the vectors $\vec{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{i} + \vec{j}$ and $\vec{v_2} = \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} = -\vec{i} + \sqrt{3}\vec{j}$

i. Find the magnitudes of $\vec{v_1}$ and $\vec{v_2}$.

ii. Find the unit vector of $\vec{v_1}$ and $\vec{v_2}$.

iii. Work out the dot product of $\vec{v_1}$ and $\vec{v_2}$ ($\vec{v_1}$. $\vec{v_2}$).

iv. Hence, find the angle between $\vec{v_1}$ and $\vec{v_2}$.

[10]

(b) Consider the following matrices:

$$A = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix} \quad B = \begin{pmatrix} -3 & -2 \\ -1 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -2 & 0 \end{pmatrix}$$

i. Write down the 2 by 2 and the 3 by 3 identity matrices, $I_{2\times2}$ and $I_{3\times3}$.

ii. Compute AB and hence write B in terms of A.

iii. Explain why CA is not defined.

iv. Work out the inverse matrix, M^{-1}, of the folowing matrix:

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}$$

[10]

(c) Let A be a 3x3 homogeneous matrix transformation corresponding to an anti-clockwise rotation about the z-axis by an angle $\frac{\pi}{2}$ and let B be a 3x3 homogeneous matrix transformation to translate the x and y coordinates by a 3 and 2 respectively.

i. Write down A, B

ii. Find the single homogeneous matrix, C, which represents transformation represented by the matrix A followed by transformation represented by the matrix B.

iii. How would the combined transformation represented by the matrix C transform the following three points which represent a triangle in the Cartesian space: (0,0), (1,1) and (1,2)?

iv. Find the matrix A^{-1} [10]