UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2016

IS51002D
Mathematical Modelling for Problem Solving

Duration: 3 hours

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and THREE questions from part B. Part A carries 40 marks, and each question from part B carries 20 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A

Multiple choice

Question 1 This question has one correct answer

(a) Which one of the following sets is a subset of $\{2,4,6,8,10,12\}$?

i. $\{14\}$

ii. $\{2,3,4\}$

iii. $\{4,8,12\}$

iv. $\{1,3,5\}$

[4]

(b) Let A, B be two subsets of a universal set U. Which of of the following describes $A \oplus B$

i. the set of elements contained in A and in B.

ii. the set of elements contained in A or in B.

iii. the set of elements contained in A but not in both.

iv. the set of elements containted in A or in B but not in both.

[4]

(c) Let A be a set of some elements. Which one of the following is correct:

i. $A \in \mathcal{P}(A)$

ii. $A \subseteq \mathcal{P}(A)$

iii. $\emptyset \subseteq \mathcal{P}(A)$

iv. None of the above

[4]

(d) Which of the following numbers is an irrational number:

i. 2.00005

ii. π

iii. $\frac{1}{2}$

iv. $3.1212\cdots$

[4]

(e) If $f(x) = 3x^2 - 2x - 5$, what is the value of f (-1)?

i. -4

ii. -10

iii. -6

iv. 0

[4]

(f) Let p be a proposition. Which one of the following is a tautology:

i. $p \vee \neg p$
ii. $p \wedge \neg p$
iii. $p \wedge T$
iv. none of the above

[4]

(g) The following sequence $1, 2, 4, 8, 16,$ is

i. arithmetic
ii. geometric
iii. neither geometric nor arithmetic

[4]

(h) The common difference, d, of the arithmetic sequence $1, 4, 7, 10, 13 \cdots$ is

i. 1
ii. 2
iii. 3
iv. 4

[4]

(i) The degree of each vertex in complete graph with n vertices is

i. n-2
ii. n-1
iii. n
iv. 2n

[4]

(j) Let A and B be two independent events. Which of of the following is correct:

i. $P(A\ and\ B) = P(A) \times P(B)$
ii. $P(A\ and\ B) = P(A) + P(B)$
iii. $P(A\ and\ B) = \frac{P(A)}{P(B)}$
iv. none of the above

[4]

Part B

Question 2 Number Systems & Sets

(a) i. Working in base 2 and showing all your working, compute the following:

$$(10101)_2 + (11011)_2 - (101)_2$$

ii. Express the hexadecimal number $(D08.1C)_{16}$ in base 2.

iii. Express the decimal number $(347)_{10}$ in base 2.

iv. Express the binary number $(110101001.011)_2$ as

- a decimal number
- a hexadecimal number
- an octal number

[10]

(b) i. Describe the set A by the listing method.

$$A = \{3r - 1 : r \in Zand\ -1 < r \leq 5\}.$$

ii. Describe the set B by the rule of inclusion method where $B = \{2, 4, 8, 16, ...1024\}$

[3]

(c) Let A and B and C be subsets of a universal set $\mathcal{U}$.

i. Draw a labelled Venn diagram depicting A, B, C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions.

ii. The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can.

[7]

Question 3 Functions

(a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d\}$ two sets. Let f be a function defined as follows:

$f : A \to B$

x	1	2	3	4	5	6
$f(x)$	a	b	a	c	d	d

i. Draw the arrow diagram to represent the function f .

ii. List the co-domain and the range of f.

iii. Find the ancestor (pre-image) of d.

iv. Show that f is not a one to one function.

v. Show that f is an onto function.

[5]

(b) Consider the function $f(x) = 2\sin 2x$.

i. What is the period of the function f?

ii. Fill in the missing values in the following table

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$2\sin 2x$					

iii. Plot the graph of f for x in $[-\pi, \pi]$.

[5]

(c) Let $f(x) = x^3 - 3x + 2$

i. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

ii. Work out the first and second derivatives of the function f (f' and f").

iii. Find all stationary points of the function f and their nature i.e. maxima, minima or inflection point.

iv. Plot the curve of the function f.

[10]

Question 4 Matrices & Transformations

(a) Given the vectors $\vec{v_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{i} + \vec{j}$ and $\vec{v_2} = \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} = -\vec{i} + \sqrt{3}\vec{j}$

i. Find the magnitudes of $\vec{v_1}$ and $\vec{v_2}$.

ii. Find the unit vector of $\vec{v_1}$ and $\vec{v_2}$.

iii. Work out the dot product of $\vec{v_1}$ and $\vec{v_2}$ ($\vec{v_1} \, . \, \vec{v_2}$).

iv. Hence, find the angle between $\vec{v_1}$ and $\vec{v_2}$.

[5]

(b) Consider the following matrices:

$$A = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix} \quad B = \begin{pmatrix} -3 & -2 \\ -1 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & -1 & 3 \\ 2 & -2 & 0 \end{pmatrix}$$

i. Write down the 2 by 2 and the 3 by 3 identity matrices, I_2 and I_3.

ii. Compute AB and hence write B in terms of A.

iii. Explain why CA is not defined.

[5]

(c) Let A be a 3x3 homogeneous matrix transformation corresponding to an anti-clockwise rotation about the z-axis by an angle $\frac{\pi}{2}$ and let B be a 3x3 homogeneous matrix transformation to translate the x and y coordinates by a 3 and 2 respectively.

i. Write down A, B

ii. Find the single homogeneous matrix, C, which represents transformation represented by the matrix A followed by transformation represented by the matrix B.

iii. How would the combined transformation represented by the matrix C transform the following three points which represent a triangle in the Cartesian space: (0,0), (1,1) and (1,2)?

iv. Find the matrix A^{-1}

[10]

Question 5 Graphs, Trees & Relations

(a) A graph with 5 vertices: a, b, c, d, e has the following adjacency list:

$a : b,\ e$

$b : a,\ c,\ d$

$c : b,\ d$

$d : b,\ c,\ e$

$e : d,\ a.$

i. Draw this graph, G.

ii. Write down the degree sequene of G. State the relationship between the number of edges in G and its corresponding degree sequence.

iii. Draw two non-isomorphic spanning trees of G

[6]

(b) A binary search tree is designed to store an ordered list of 5000 records, numbered 1,2,3,...,5000 at its internal nodes.

i. Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

ii. What is the height of this tree?

[4]

(c) Given S be the set of integers $\{5, 6, 7, 8, 9, 10\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,
for all $x, y \in S$, xRy if $(x - y)$ is a multiple of 3.

i. Draw the digraph of $\mathcal{R}$.

ii. Say with reason whether or not $\mathcal{R}$ is

- reflexive;
- symmetric;
- anti-symmetric;
- transitive.

In the cases where the given property does not hold provide a counter example to justify this.

iii. is R a partial order? Explain your answer

iv. is R an equivalence relation? If the answer is yes, write down the equivalence classes for this relation.

[10]

Question 6 Logic, Sequences & Probability

(a) Let p and q be the following propositions:

p : *'this object is a triangle'*
q : *'this object is blue'.*

i. Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

"this object is a blue triangle"
"if this object is blue then it is a triangle"
"this object is not blue or is a triangle, but not both".

ii. Construct the truth table for the statement $q \to p$.

iii. Write in words the contrapositive of the statement given symbolically by "$q \to p$".

[7]

(b) Let the sequence u_n be defined by the recurrence relation

$$u_1 = 1 \ \ and \ \ u_{n+1} = u_n + 2n, \ \ for \ n \geq 1.$$

i. Calculate u_2, *and* u_3, showing all your working.

ii. Prove by mathematical induction that the *nth* term, where $n \geq 1$, is given by

$$u_n = n^2 - n + 1.$$

[6]

(c) In an experiment a coin is tossed three times and each time it is noted whether the coin comes up heads (H) or tails (T). The final result is recorded as an ordered triple, such as (H,H,T). Let A be the event that the last toss comes up as a tail and B be the event that there is only one tail in the triple.

i. Draw a rooted tree to model this process.

ii. Calculate the probabilities of the events A, B, $A \cap B$ and $A \cup B$.

iii. Are A and B independent events? Justify your answer.

[7]