

## Formulae sheet

### The quadratic formula

The solutions of  $ax^2 + bx + c = 0$  where  $a \neq 0$  are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Sum of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

### Sum of a geometric series

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

### Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

### Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Question 1** This question is about order of operations, percentages, standard form and significant figures

a) Evaluate the following expressions:

i.  $(3 - 1)^2 + 5 - 2 + 1$

ii.  $3 - 1^2 + 5(-2 + 1)$

[2]

b) Convert

i.  $\frac{7}{5}$  to a percentage

ii. 20% to a fraction in its simplest form

[2]

c) Fred, Greg and Habib invest in the ratio 3:5:8. Habib's share is £100,000. What is the total investment made?

[1]

d) Round the following numbers

i. 2.30355 to 2 decimal places

ii. 780.535 to 2 significant figures

[2]

e) The length of a field is measured as 200m to the nearest metre. What is the greatest length the field could be?

[1]

f) Calculate  $(1.081 \times 10^2) \div (4.7 \times 10^{-1})$ . Give your answer in standard form.

[2]

**Question 2** This question is about algebraic expressions and substitution

a) Expand the following expressions.

i.  $(x - y)^2$

ii.  $(2y - x)(x - y)^2$

[2]

b) Evaluate the following expressions when  $x = 1$  and  $y = 2$

i.  $(x - y)^2$

ii.  $(2y - x)(x - y)^2$

[2]

c) Simplify the following expressions, give answers in their simplest form

i.  $\frac{x+2}{3x+6}$

ii.  $\frac{x^2-4}{x+2}$

[2]

d) Simplify the following expressions, give answers as a single fraction in its simplest form

i.  $\frac{2x}{9(x+1)} - \frac{1}{3}$

ii.  $\frac{2x}{9(x+1)} \div \frac{1}{2x+2}$

[2]

e) Simplify the following expressions.

i.  $(b^5cd) \times (d^{-2}c)$

ii.  $(a^{-2}b^0)^2$

[2]

**Question 3** This question is about number bases, and factors and multiples

- a) Convert the following numbers to decimal
- i.  $100100111_2$
  - ii.  $765_8$
- [2]
- b) Convert the decimal number  $176_{10}$
- i. to base 2
  - ii. to base 5
- [2]
- c) Convert the octal number  $771_8$  to:
- i. binary
  - ii. hexadecimal
- [2]
- d) Calculate the following:
- i.  $10011_2 - 1110_2$ , give your answer in binary
  - ii.  $3B2_{16} + 27B_{16}$ , give your answer in hexadecimal
- [2]
- e) Find
- i. the highest common factor of 48 and 80
  - ii. lowest common multiple of 48 and 80
- [2]

**Question 4**      **This question is about logic and sets.**

a)

- i. Construct and complete a truth table for the following logical expression.

$$(P \rightarrow Q) \vee (P \wedge \neg Q)$$

[4]

- ii. State whether  $(P \rightarrow Q) \vee (P \wedge \neg Q)$  is a tautology, a contradiction or a contingency

[1]

b)  $A, B,$  and  $C$  are subsets of a universal set  $\mathcal{E}$  as follows:

$$\mathcal{E} = \{x: x \text{ is an integer and } 20 < x \leq 30\}$$

$$A = \{x: x \text{ is a multiple of } 5\}$$

$$B = \{x: x \text{ is a multiple of } 8\}$$

$$C = \{x: x \text{ is not divisible by } 2\}$$

List the following sets:

i.  $\overline{A \cup B}$

ii.  $A \cap \bar{B}$

iii.  $\overline{A \cup B} \cap B$

[3]

c) Draw and shade Venn diagrams to represent the following general sets

i.  $\overline{A \cup B}$

ii.  $A \cap \bar{B}$

[2]

**Question 5**                      **This question is about linear, simultaneous and quadratic equations**

a) Solve the following equations.

i.  $2x + 5 = 4x - 9$

ii.  $2(x + 5) = 4(x - 9)$

[2]

b) Solve the following simultaneous equations.

$$\begin{cases} 5c + 2d = 8 \\ 4c + 3d = 5 \end{cases}$$

[2]

c) Factorize the following

i.  $5x^2 + 15x$

ii.  $2x^2 - 32$

iii.  $x^2 - 3x - 10$

[3]

d) Hence or otherwise solve the following quadratic equations

i.  $5x^2 + 15x = 0$

ii.  $2x^2 - 32 = 0$

iii.  $x^2 - 3x - 10 = 0$

[3]

**Question 6**      **This question is about sequences and series.**

a) Given the sequence  $u_i = 5(i - 2)$        $i = 1, 2, 3 \dots$

- i. Find  $u_1, u_2, u_3$  and  $u_{100}$
- ii. State whether the sequence  $u_i$  is an arithmetic progression, a geometric progression or neither

[3]

b) Find the number of terms in the following sequence:

$$128, 64, 32 \dots \frac{1}{16}$$

[1]

c) Find the sum of the following series,  
(you may use your answer to b) above)

$$128 + 64 + 32 + \dots + \frac{1}{16}$$

[2]

d) Write out the following sum in full

$$\sum_{i=0}^3 (2^i + 2i)$$

[2]

e) Write the following using sigma notation

$$2 + 4 + 6 + \dots + 2(n - 5)$$

[2]

**Question 7**      **This question is about functions and matrices**

Consider the following functions:

$$f(x) = 2x - 1 \qquad g(x) = x^2 - 1 \qquad h(x) = \frac{1}{x} - 1, x \neq 0$$

a) Evaluate the following.

- i.  $g(2)$
- ii.  $h(g(2))$

[2]

b) Find an expression for:

- i.  $g(2x)$
- ii.  $g(f(x))$

[2]

c) Find the inverse functions

- i.  $f^{-1}(x)$
- ii.  $h^{-1}(x)$

[2]

d) Given the following matrices  $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$      $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- i. Find the product  $AB$
- ii. State the relationship between  $A$  and  $B$

[3]

e) Given the following matrix  $M$

$$M = \begin{pmatrix} x & 2 \\ 2 & 1 \end{pmatrix}$$

Find  $x$  given that  $M$  has no inverse.

[1]



**Question 8**      **This question is about trigonometry**

**Give your answers to the nearest degree or to 2 significant figures**

a) Triangle  $PQR$  is a right angled triangle with angle  $R = 90^\circ$  and side  $r = 10m$  and side  $p = 5m$

- i. Find the length of side  $q$
- ii. Find angle  $Q$

[2]

b) In the triangle  $ABC$  angle  $A = 55^\circ$  side  $a = 48mm$  and  $c = 22mm$

- i. Find angle  $C$
- ii. Find the length of side  $b$

[3]

c) In the triangle  $DEF$  angle  $D = 98^\circ$  side  $e = 63mm$  and  $f = 39mm$   
Find the length of side  $d$

[1]

d)

- i. Draw the graph of  $y = \sin 2x$  for  $0^\circ \leq x \leq 360^\circ$ , show where it cuts the axes
- ii. Use your graph to find all the values of  $x$  between  $0^\circ$  and  $+ 360^\circ$  for which  $\sin 2x = -0.5$

[4]

**Question 9**      **This question is about graphs**

- a) Plot the graph of  $y = 3 - x$  for  $-3 \leq x \leq 5$  [1]
- b) On the same grid plot the graph of  $y = x^2 - 2x$  for  $-3 \leq x \leq 5$  [2]
- c) Use your graphs to find solutions to the following simultaneous equations. Show your method clearly on the graph.

$$\begin{cases} y = 3 - x \\ y = x^2 - 2x \end{cases}$$

[2]

- d)
- Find the equation of the line that passes through  $(2, -4)$  and  $(6, 4)$ .
  - State its gradient
  - State the y-intercept
- [3]
- e) Draw a graph of the function  $y = 2^{x+1}$  for  $-10 \leq x \leq 4$   
Show clearly where it cuts the axes and mark any asymptotes

[2]

**Question 10      This question is about probability**

a) You have two fair dice. You toss both dice and find the difference between the numbers

- i. Draw a space diagram (table) to represent the possible outcomes
- ii. Find the probability of getting an outcome of:
  - i) 0
  - ii) 1 or 2

[3]

b) A biased 6 sided dice is thrown. The probability of scoring 1 is 0.2, and the probability of scoring 2 is 0.3. The scores 3 to 6 all have equal probability

- i. What is the probability of scoring 2 or above
- ii. What is the probability of scoring 6
- iii. What is the probability of scoring 12

[3]

c) You have letter cards that spell ADDITION. You pick 2 cards without replacement. Find the probability of:

- i. The first card is the letter D
- ii. The second card is the letter D
- iii. Both cards are D
- iv. Both cards are the same
- v. Both cards are different

[4]

**END OF EXAMINATION**