# UNIVERSITY OF LONDON 

GOLDSMITHS COLLEGE

## B.Sc. Examination 2015

DEPARTMENT OF COMPUTING

## IS53002A Neural Networks

Duration: 2 hours 15 minutes
Date and time:

There are five questions on this paper. You should answer no more that THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 75 marks available on this paper.
Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

## THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

## Question 1.

a) Describe briefly how a batch training algorithm for neural networks operates. What is the alternative kind of training algorithm for neural networks and how does this alternative algorithm process the training examples? [7]
b) Define the two most commonly used activation functions in multilayer Perceptron networks. [4]
c) Give the expression for calculating the total number of weights and thresholds in a two-layer multilayer Perceptron (MLP) network with $z$ inputs, $H$ hidden nodes and $k$ outputs. [4]
d) Consider a Radial-basis function (RBF) network with 4 neurons each having Gaussian basis functions. Assume that the initial weight vector is:
$\boldsymbol{w}=(0.1,-0.3,0.2,-0.15)$, the basis function variances are: $s^{2}=(0.22,0.33,0.44,0.11)$, and the corresponding centres are as follows: $\boldsymbol{c}_{1}=(0,1,0,1), \boldsymbol{c}_{2}=(1,0,1,1)$, $\boldsymbol{c}=(1,1,0,0)$ and $\boldsymbol{c}_{4}=(1,0,0,1)$.
i) Show the analytical formula for computing the output of this RBF network including the calculations performed in each network node. [6]
ii) Compute the RBF network output with the following training input vector $\mathbf{x}=(1,1,0,1)$, with precision up to and including the fourth digit after the decimal point. [4]

## Question 2.

a) Explain how classification into $\mathrm{K}(\mathrm{K}>2$ ) classes can be performed using a single-layer Perceptron network which infers a linear function? [4]
b) Give the formula for offline (batch) gradient descent training of unthresholded Perceptron networks. Explain each term in the training formula. [5]
c) Consider a single-layer Perceptron neural network with five inputs and no threshold. This Perceptron computes the sum: $s=w_{1} x_{1}+w_{2} x_{2}+w_{3} x_{3}+w_{4} x_{4}+w_{5} x_{5}$, and passes it next through the discrete activation function: Threshold ( $s$ ) = 1 if $s>0$. Assume that the learning rate is one. Demonstrate one cycle of training this Perceptron using the following example vectors, provided sequentially:

| $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ | $y$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 |

Begin with the following initial weights: $\left(w_{1}, w_{2}, w_{3}, w_{4}, w_{5}\right)=(0,0,0,1,1)$. Show the network output and compute the weights changes after each example. [16]

## Question 3.

a) Define the gradient descent training rule for multilayer networks with weight decay regularization. Explain the main components of the rule. [5]
b) Consider a multilayer neural network with two nodes: one hidden and one output using sigmoidal activations (that is, $S_{1}$ and $S_{2}$ denote sigmoidal activation functions) given in the figure below. There are two inputs passed to the network: $\left(x_{1}, x_{2}\right)$, and six weights: ( $w_{1}, w_{2}, w_{3}, w_{4}, w_{5}, w_{6}$ ). Both of the hidden neurons have bias connections $w_{4}$ and $w_{6}$ which are fixed at 1 .


Demonstrate the operation of the backpropagation training algorithm on this network including:
i) Develop the expressions for computing the error and weight updates $w_{3}^{\prime}, w_{5}^{\prime}$ and $w_{6}^{\prime}$ for the connections entering the output node. Explain the meaning of each term in the expressions. [10]
ii) Develop the expressions for computing the backpropagated errors and weight updates $w_{1}^{\prime}, w_{2}^{\prime}$, and $w_{4}^{\prime}$ for the connections entering the hidden node. Explain the meaning of each term in the expressions. [10]

## Question 4.

a) Explain briefly what operations are performed in each of the three main phases of the training algorithm for self-organizing Kohonen networks. [9]
b) Draw a picture of a self-organizing Kohonen network with three neurons. [6]
c) Let a self-organizing Kohonen neural network with two neurons be given. There are four inputs passed to each neuron. Assume that the initial weight vectors are: $\boldsymbol{w}_{1}=(0.15,0.2,-0.33,0.4)$, and $\boldsymbol{w}_{2}=(0.25,-0.1,0.44,-0.33)$.
i) Compute the summation block using the input vector:
$\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=(0.1,0.4,0.3,0.2)$ and determine the index of the largest component (neuron) in the summation block. [4]
ii) Train the neuron computed in part (i) and show the weight updates using learning rate $\eta=0.25$. [6]

## Question 5.

a) i) Describe what operations are involved in the retrieval phase of the Hopfield neural networks. [5]
ii) Define the formula for updating of the state in Hopfield neural networks after a testing probe vector is presented to the network during the retrieval phase. [5]
c) Consider a Hopfield neural network with 4 neurons and 4 inputs: $x_{1}, x_{2}, x_{3}, x_{4}$. The initial weights are given by the following matrix:

$\mathbf{W}=$| 0 | -0.1 | 0.2 | 0.4 |
| :---: | :---: | :---: | :---: |
| -0.1 | 0 | -0.15 | 0.3 |
| 0.2 | -0.15 | 0 | -0.2 |
| 0.4 | 0.3 | -0.2 | 0 |

i) Using the state $[0,1,0,1]$ compute the new state after neuron 1 fires. [4]
ii) Assuming that the network operates in synchronous mode, that is starting from the updated state, compute the next state after neuron 2 fires. [4]
iii) Starting from state $[1,0,1,1]$ the output of neuron 2 is 1 while it should be 0 . Using the Widrow-Hoff rule train the weights of the network to correct the situation. [7]

