

Question 1 This question is about number systems

a)

- i. Convert the decimal number 103_{10} to binary. **Show all your working**
- ii. Write the binary number 101100.101_2 in expanded form and hence find its decimal equivalent
- iii. Convert the hexadecimal number $A5.7_{16}$ into binary. **Show all your working**
- iv. Hence or otherwise convert the hexadecimal number $A5.7_{16}$ number into octal.

[6]

b) Working in binary perform the following calculation $100010_2 - 1101_2$

[2]

c) Convert the recurring decimal number $0.429429\dots$ to a fraction in its simplest form.

[2]

Question 2 **This question is about set theory**

- 1.
- a.
- i. Describe the following set by the listing method
 $\{2^n: n \in \mathbb{Z} \text{ and } 3 \leq n \leq 6\}$
- ii. Describe the following set by giving a suitable universal set and rules of inclusion $\{3, 5, 7, 9\}$

[3]

- b. Let $X = \{a, e, i\}$ and $Y = \{a, b, c, d, e\}$ be subsets of a universal set $U = \{a, b, c, d, e, f, g, h, i, j\}$. Find the following

- i. $\bar{X} \cap \bar{Y}$
ii. $\overline{X \cup Y}$

[3]

- c. Let A, B and C be subsets of a universal set U

- i. Construct a membership table for the set $A \cap \overline{B \cup C}$
- ii. By using membership tables or otherwise show that
 $A \cap \overline{B \cup C} = A \cap \bar{B} \cap \bar{C}$.

[4]

Question 3 This question is about trigonometric and exponential functions

Give your answers to 1 decimal place or the nearest degree.

a. ABC is a right angled triangle with $A = 90^\circ$, $AB = 10\text{cm}$, $BC = 23\text{cm}$

- i. Find length AC
- ii. Find angle B

[2]

b. FGH is a triangle with $F = 122^\circ$, $GH = 23\text{cm}$, $G = 43^\circ$. Find length FH

[2]

c. XYZ is a triangle with $XY = 15\text{cm}$, $XZ = 21\text{cm}$, $YZ = 18\text{cm}$. Find angle Y

[2]

d.

- i. By completing a table of values or otherwise, sketch the function $f(x) = \cos x$ for $x \in [-360^\circ, 360^\circ]$
- ii. Find all the values of $x \in [-360^\circ, 360^\circ]$ for which $\cos x = 0.3$

[4]

Question 4 **This question is about graph sketching**

a. Given the function $f(x) = 2x + 3$

- i. Find the gradient
- ii. Find the y-intercept
- iii. Draw the graph of the function
- iv. Draw the graph of the inverse function $f^{-1}(x)$

[5]

b. Find the minimum value of the function $g(x) = (x - 4)(x + 1)$

[2]

c. Sketch the function $h(x) = x(x + 2)(x + 3)$

[3]

Question 5 **This question is about the binomial theorem, calculus and velocity and acceleration**

- d.
- i. Write out the first four terms of following binomial expansion. (You may use the formula)
 $(1 + x)^8$
[2]
 - ii. Find the value of the 1.003^8 to 2 decimal places by using the binomial expansion of $(1 + x)^8$ with an appropriate value of x . You may use your answer to part a. above
[2]
- e. Find $\frac{dy}{dx}$ the gradient function of $y = x^5 + 3x + 2$
[2]
- f. The acceleration of a particle is given by $a = 3t^2 + 2$, where a is the acceleration in ms^{-2} and t is the time in seconds
- i. Use your expression for the acceleration to find the acceleration of the particle when
 1. $t = 0$ seconds
 2. $t = 2$ seconds[2]
 - ii. Which of the following could give the velocity of the particle
 $v = 6t + 2, v = 6t, v = t^3 + 2t, v = t^3 + 2t + 2$
[2]

Question 6 **This question is about vectors and matrices**

- a. Given the vectors $\mathbf{u} = \begin{pmatrix} 1 \\ \sqrt{5} \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ \sqrt{5} \\ 2 \end{pmatrix}$
- Write \mathbf{u} in terms of the unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k}
 - Find the magnitudes of \mathbf{u} and \mathbf{v}
 - Compute $\mathbf{u} \cdot \mathbf{v}$, the dot product of \mathbf{u} and \mathbf{v}
 - Find the angle between \mathbf{u} and \mathbf{v}

[7]

b.

- Find the determinant of the matrix $M = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$
- State the determinant of M^{-1}

[3]

Question 7 **This question is about matrix transformations**

a. Given matrices

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

- i. Describe the transformations represented by A and B
[2]
- ii. Calculate BA the product of the matrices A and B
[2]
- iii. A triangle has vertices $(1, 1)$ $(1, 2)$ and $(2, 1)$. Find the vertices of the triangle after it has been transformed by matrix A followed by matrix B
[2]

b.

- i. Find the 3 by 3 matrix R which represents a rotation of 90° anticlockwise about the origin
[2]
- ii. Given that R represents a rotation of 90° anticlockwise about the origin, describe the single transformations represented by
 1. R^2
 2. R^{-1}[2]

Question 8 **This question is about complex numbers**

- a. Given complex numbers $z_1 = -1 + i$ and $z_2 = 1 + 2i$
- i. Find $z_1 \times z_2$, give your answer in the form $a + ib$ [1]
 - ii. Find $z_1 \div z_2$, give your answer in the form $a + ib$. You may use the complex conjugate [1]
 - iii. Convert z_1 to polar form [2]
 - iv. Hence convert z_1 to exponential form [1]
- b. Given complex numbers $z_1 = 2e^{\frac{\pi i}{4}}$ and $z_2 = \frac{1}{2}e^{-\frac{\pi i}{3}}$
- v. Find $z_1 \times z_2$, give your answer in exponential form [1]
 - vi. Find z_1^3 , give your answer in exponential form [2]
 - vii. One possible value of $z_1^{\frac{1}{3}}$ is $\sqrt[3]{2}e^{\frac{\pi i}{12}}$. Find the remaining values of $z_1^{\frac{1}{3}}$. Give your answers in exponential form [2]

END OF EXAMINATION