UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2015

IS51002C-Resit
Mathematical Modelling for Problem Solving

Duration: 2 hours 15 minutes

Date and time:

There are Six questions in this paper. You should answer no more than THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] *brackets.*

There are 75 marks available on this paper.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) The first 16 integers ≥ 0 can be represented by 4-bit binary strings.

i. List these integers in hexadecimal, together with their binary equivalents.

ii. Find the hexadecimal equivalent of the binary numeral 100101.01 and find the binary equivalent of the hexadecimal numeral $59.A$

[5]

(b) Working in the binary system compute the following operation, showing all your working:

$$(10110)_2 + (11011)_2 - (111)_2$$

[4]

(c) i. Define what is meant by an irrational number.

ii. Showing all your working, express the repeating decimal 0.270270.....
as a fraction in its simplest terms.

[4]

(d) Let $A = \{2n : n \in \mathbb{Z}^+\}$ and $B = \{3, 6, 9, 12, ...\}$ be two sets of numbers.

i. Describe the set A by the listing method.

ii. Describe the set B by the rules of inclusion method.

iii. Find the two sets $A \cap B$ and $A - B$, by the listing method. [6]

(e) Let P, Q and R be subsets of a universal set $\mathcal{U}$.

i. Construct a membership table for the set $X = P' \cup (Q \cap R)$.

ii. Draw a labelled Venn diagram showing $P, Q,$ and R intersecting in the most general way.

iii. Shade the region X on your diagram.

iv. Is the set $P' \cap R \subseteq X$? Justify your answer. [6]

Question 2

(a) Let n be an element of the set $\{$1,2,3,4,5,6,7,8,9$\}$ and p and q be the following statements about n:

p: $n \leq 5$
q: n is even.

i. Express each of the three following compound propositions symbolically by using p, q and appropriate logical symbols.

n>5 and n is odd.
if n≤5 then n is odd.
n>5 or n is even, but not both.

[3]

ii. Draw up the truth tables for the following statements and find the values of n for which they are true:

$$p \vee \neg q; \qquad \neg p \wedge q$$

[4]

iii. Use the truth table to find a statement that is logically equivalent to $\neg p \rightarrow q$.

[3]

(b) Let p and q be the two propositions defined in (a).

i. Write the contrapositive of the statement:

if n is even then $n \leq 5$.

[3]

ii. Write the result in (i) into it's equivalent logical expression.

[2]

(c) i. Construct and draw a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output $(p \wedge q) \vee \neg q$. Label all the gates appropriately and also give labels to show the output from each gate. [4]

ii. Construct a logic table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q. [3]

iii. Show that $(p \wedge q) \vee \neg q$ is equivalent to $q \rightarrow p$. [3]

Question 3

(a) A function $f : X \to Y$, where $X = \{p, q, r, s\}$ and $Y = \{1, 2, 3, 4, 5\}$, is given by the subset of $X \times Y$, $\{(q, 3), (r, 3), (p, 5), (s, 2)\}$.

i. Show f as an arrow diagram.

ii. State the domain, co-domain and range of f.

iii. Say why f does not have the one-to-one property and why f does not have the onto property, giving a specific counter example in each case.

[10]

(b) Let $f : \mathbb{R} \to [-3, \infty[$ with $f(x) = x^2 - 3$

i. Plot the function f for x in [-3,3].

[2]

ii. Show that f is not invertible.

[3]

(c) i. State the condition to be satisfied in order for a function to have an inverse.

ii. Given the function $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2x - 1$.

1. Show that f is a one to one function.
2. Show that f is an onto function.
3. Find the inverse inverse function f^{-1}.

[5]

iii. Let g be a function defined as follows:
$g : \mathbb{Z} \to \mathbb{R}$ where $g(x) = 2x - 1$.

1. Is g a one to one function? Explain your answer.
2. Is g an onto function? Explain your answer.
3. Is g invertible? Explain your answer.

[5]

Question 4

(a) i. Draw a binary tree to store a list of 14 records.

[6]

ii. What is the maximum number of comparisons that would have to be made in order to locate an existing record from this list of 14 records?

[2]

iii. Find the height of binary search tree to store a list of 4000. records numered 1,2,...4000 at its internal nodes.

[2]

(b) i. Use the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to find a formula for $s_n = \sum_{k=1}^{n}(3k+1)$ in terms of n. Use this formula to find this sum when $n = 10$.

[3]

ii. Write the following expression in $\sum$ notation appropriate limits and calculate its value.

$$50 + 51 + 52 + ... + 100$$

[2]

(c) i. Given the following sequence

$$1, 4, 7.10, 13, 16, 19, \cdots$$

1. Is this sequence arithmetic of geometric sequence? If you identify it as arithmetic, specify the common difference d. If you identify it as geometric, specify the common ratio r.
2. In terms of n find an expression for the sum of the first n terms of this sequence.
3. Find the sum of the first 10 terms.

[5]

ii. Let u_n be the sequence of numbers defined by

$$u_1 = 0; \text{ and}$$
$$u_{n+1} = u_n + n$$

1. Calculate u_2 and u_3.
2. Prove by induction that

$$u_n = \frac{n(n-1)}{2} \qquad \text{for all } n \geq 1$$

[5]

Question 5

(a) Let G be the simple graph with vertex set $V(G) = \{a, b, c, d, e\}$ and adjacency matrix

$$\mathbf{A} = \begin{array}{c} \\ \text{a} \\ \text{b} \\ \text{c} \\ \text{d} \\ \text{e} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{a} & \text{b} & \text{c} & \text{d} & \text{e} \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

i. Say how the number of edges in G is related to the entries in the adjacency matrix $\mathbf{A}$ and calculate this number.

ii. Draw G.

iii. Find a spanning tree T_1 for G and give its degree sequence.

iv. Find a spanning tree T_2 for G which is **not** isomorphic to T_1 and give a reason why it is not isomorphic.

[10]

(b) i. Given the vectors $\vec{v_1} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $\vec{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

1. Find the magnitude of $\vec{v_1}$ and $\vec{v_2}$
2. Compute the dot production of $\vec{v_1}$ and $\vec{v_2}$
3. Find the angle between $\vec{v_1}$ and $\vec{v_2}$.

[3]

ii. Which of the following homogeneous coordinates (2,6,3), (4,6,2), (2,4,1) and (8,12,4) represent the point (2, 3)?

[2]

(c) Let A be a 3x3 homogeneous matrix transformation corresponding to an anti-clockwise rotation about the z-axis by an angle $\frac{\pi}{2}$ and let B be a 3x3 homogeneous matrix tranformation to scale the x and y coordinates by a factor 3 and 2 respectively

i. Write down A, B

ii. Find the single homogeneous matrix, C, which represents transformation represented by the matrix A followed by transformation represented by the matrix B.

iii. How would the combined transformation represented by the matrix C transform the following three points which represent a triangle in the Cartesian space: (0,0), (-1,1) and (1,1)?

iv. Find the matrix A^{-1} [10]

Question 6

(a) i. What properties should a graph have in order for it to be:

1. a simple graph;
2. a complete grqph

[2]

ii. Say why is it imposible to construct a simple graph with the following degree sequence:

$$5, 4, 3, 3, 2$$

iii. Is it possible to construct a 3-regular graph with 5 vertices? Explain your answer.

[3]

(b) Given the graph G with vertices $v_1,\ v_2, ...\ v_7$ and adjacency list

$v_1 : v_2, v_4$

$v_2 : v_1, v_3$

$v_3 : v_2, v_4$

$v_4 : v_1, v_3, v_5$

$v_5 : v_4, v_6$

$v_6 : v_5, v_7$

$v_7 : v_5, v_6.$

i. Draw the graph of G.

[2]

ii. What is the degree sequence of G? Find the number of edges in G.

[2]

iii. Say how many edges there are in a tree with n vertices. Hence explain how many edges must be removed from G to create a spanning tree.

[3]

iv. Draw all non isomorphic trees of G.

[3]

(c) Given S be the set of integers $\{1, 2, 5, 6, 7, 8, 9\}$.

i. Let $\mathcal{R}_1$ be a relation defined on S by the following condition such that, for all $x, y \in S$, $x\mathcal{R}_1 y$ if $x \leq y$.

1. Draw the digraph of $\mathcal{R}_1$.
2. Show that $\mathcal{R}_1$ is a total order.

[5]

ii. Another relation, $\mathcal{R}_2$, is defined on S as follows:

$$\text{for all} x, y \in S,\ x\mathcal{R}_2 Y \text{ if } (x + y)\ mod\ 2\ =\ 0$$

1. Draw the digraph of this relation on S.
2. Explain why this relation is an equivalence relation but not a partial order.
3. Write down the equivalence classes for this relation. [5]