UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2014

IS50003B
FOUNDATIONS OF PROBLEM SOLVING

Duration: 2 hours 15 minutes

Date and time:

There are five questions in this paper. You should answer no more than THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] *brackets.*

There are 75 marks available on this paper.

THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM

Question 1

(a) i. Given the following array of integers:

24	8	21	25	50	40	65	73	81

(1) What is the maximum number of comparisons it will take the `linear search` algorithm to find a value in this array?

(2) Write down the complexity of the linear search algorithm.

(3) Can a `binary` search algorithm be used to search for data in this array? Explain your answer.

ii. Write a pseudo-code for the binary search algorithm.

[13]

(b) Given the following array of integers:

1	10	25	33	45	56	65	84	91

i. Using the `binary search` algorithm, describe the sequence of steps it would take to find the number 56.

ii. What is the maximum number of comparisons it will take the `binary search` to find data in this array?

iii. What is the maximum number of comparisons it will take the `binary search` to find a value in an array of n elements?

[12]

Question 2

(a) i. Use `bubble_sort` algorithm to rearrange the following numbers into ascending order, showing the list of numbers after each pass.

$$5 \quad 1 \quad 2 \quad 6 \quad 9 \quad 4 \quad 3$$

ii. Write down the number of comparisons and swaps on each pass.

iii. Find the maximum number of comparisons and the maximum number of swaps that might be needed in a `bubble_sort` to rearrange seven numbers into ascending order.

[13]

(b) i. Describe how the `selection_sort` algorithm works.

ii. Use the `selection_sort` algorithm to rearrange the following number into ascending order, showing the list of numbers.

$$35 \quad 65 \quad 30 \quad 60 \quad 20$$

iii. Using the `selection_sort` algorithm, find the maximum number of comparisons needed to sort a list of five elements into ascending order. Can you generalise this for a list of n elements?

[12]

Question 3

(a) The following list represents the lengths in minutes of nine radio programmes. They are to be recorded onto tapes. Each tape stores up to 100 minutes of programmes.

$$29, 52, 73, 87, 74, 47, 38, 61, 41$$

i. Find the lower bound for the number of tapes needed to store all the above nine programmes.

ii. Use the `first_fit` bin packing algorithm to fit the programmes onto tapes.

iii. Use the `first_fit_decreasing` algorithm to fit the nine programmes.

[12]

(b) Five tour guides, Adam, Emiley, Gareth, Rob and Wendy need to be assigned five coach trips, 1, 2, 3, 4, and 5. Preferences of each tour guides is shown by a bipartite graph in Figure 1.

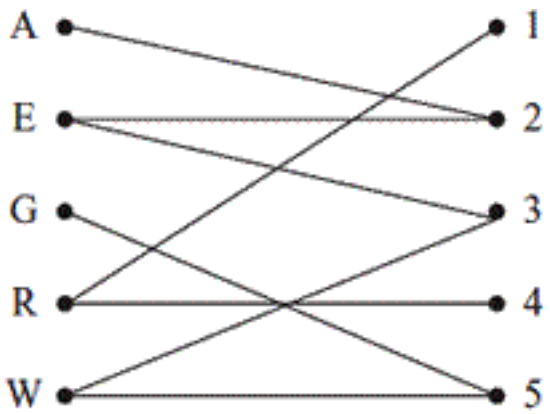


Figure 1

i. Starting with the initial state given in Figure 2, use the maximum matching algorithm to increase to number of matching. State the alternative path you used.

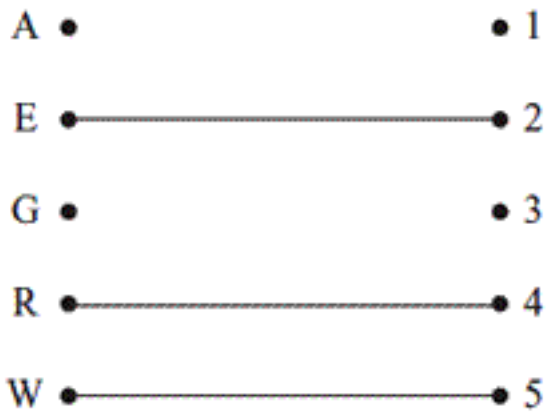


Figure 2

ii. List the improved matching found in (i).

iii. A complete matching in this case is not possible. Explain why?

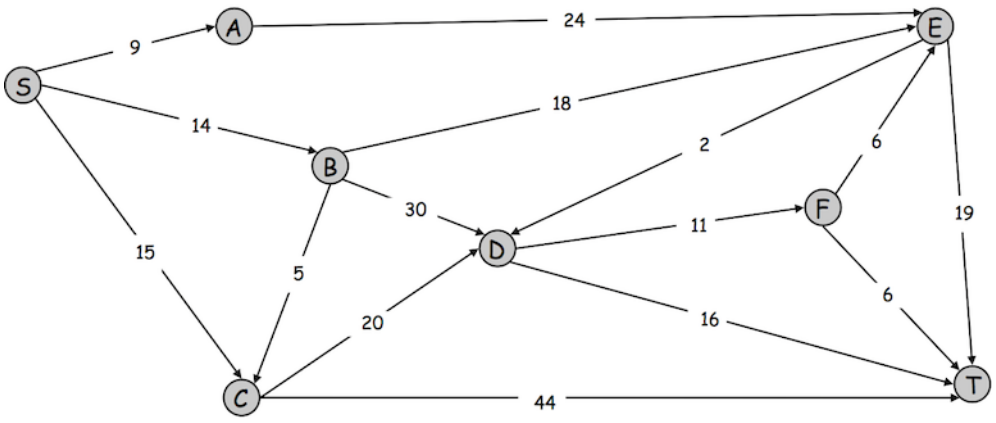
iv. Suppose Wendy agrees to be assigned to coach trip 3, 4 or 5. Starting with your current maximal matching, use the maximum matching algorithm to obtain a complete matching.

[13]

Question 4

(a) i. Explain how Dijkstra's algorithm works.

ii. Given the following directed graph:



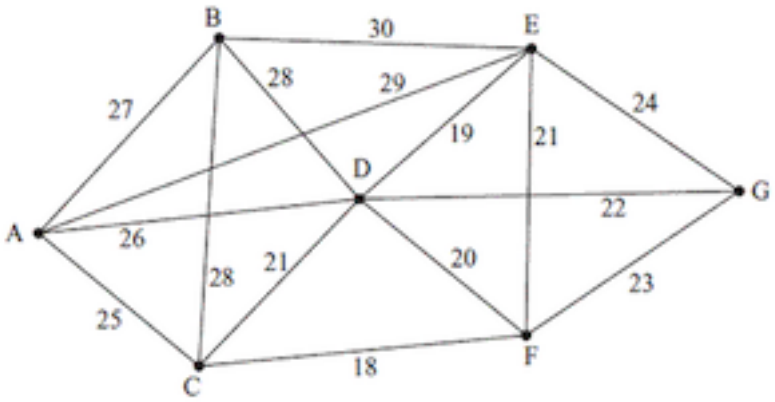
Use Dijkstra's algorithm to find the shortest route from S to T. Explain your answer.

[13]

(b) i. Explain how `Kruskal's` algorithm works.

ii. Using Kruskal's algorithm, find a minimum spanning tree (MST) of the following weighted graph.

iii. What is the difference between Kruskal's and Prim's algorithm?



[12]

Question 5

(a) Given the following system of linear equations:

$$\begin{aligned} x - 2y + z &= 1 \\ x + y + z &= 10 \\ 3x + 2y - 2z &= 2 \end{aligned}$$

i. Write down the augmented matrix.

ii. Use Gaussian elimination to solve this system.

[10]

(b) Consider the following linear programming problem. The objective is to maximise the value of the variable $p = 6x + 2y$.

$$\text{Subject to } \begin{cases} 2x + y \leq 10 \\ x + y \leq 8 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

i. Graph the feasible region for the problem.

ii. Find the optimal solution.

[15]