UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B.Sc. Examination 2013

COMPUTING AND INFORMATION SYSTEMS

IS53002A Neural Networks

Duration: 2 hours 15 minutes

Date and time:

There are five questions on this paper. You should answer no more that THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 75 marks available on this paper.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1.

a) i) Explain briefly which of the following artificial neural networks can learn the Boolean bivariate XOR function: the single-layer Perceptron with a linear activation function, the multilayer Perceptron and the Radial-basis function network. **[6]**

ii) Give the most commonly used activation functions in multilayer Perceptron networks. **[4]**

b) Let a Radial-basis function (RBF) network with 3 neurons and Gaussian basis functions be given. Suppose that the initial weight vector is: $\boldsymbol{w} = (-0.1, 0.2, 0.3)$, the basis function variances are: $\boldsymbol{s}^2 = (0.45, 0.4, 0.35)$, and the corresponding centres are as follows: $\boldsymbol{c}_1 = (1,1,0)$, $\boldsymbol{c}_2 = (1,0,0)$ and $\boldsymbol{c}_3 = (0,0,1)$. Assume that the following boolean input vector is applied to this network: $\mathbf{x} = (1,0,1)$.

i) Give the analytical formula for computing the output of this RBF network. **[3]**

ii) Compute the network output with the given Boolean input vector with precision up to and including the fourth digit after the decimal point. **[5]**

c) Design a single-layer Perceptron network with 3 binary inputs and a threshold, each associated with a corresponding weight, that implements the Boolean majority function. Given suitable weights the majority function should produce target $y=1$ only when most of the inputs are one, that is these $x_i=1$ are more than these $x_i=0$ $(0<=i<=3)$. Use the thresholding function: $f(s)=1$ if the weighted summation of the inputs is $s>0$ and 0 otherwise.

i) Define the analytical formula for the majority function using weights w_i and inputs x_i. **[3]**

ii) Draw the network and suggest suitable positive integer weights without training the network. **[4]**

Question 2.

a) i) Define the batch (offline) gradient descent training algorithm for unthresholded Perceptron networks. Explain each term in the training formula. **[5]**

ii) Explain briefly what is the difference between batch (offline) and incremental (online) training of unthresholded Perceptrons? **[4]**

b) Discuss briefly how the performance of a single-layer thresholded Perceptron with two inputs will change if we multiply all the weights and the threshold by a negative constant z. Will there be a change in the performance if we multiply them by 0? **[6]**

c) Perform (offline) training of a single-layer Perceptron with 2 inputs without bias, and activation function defined by: $Threshold(s)=1$ if $s>=0$ and (-1) otherwise. Use the following modified weight update rule: $\Delta\boldsymbol{w} = \Delta\boldsymbol{w} + 0.3 * \boldsymbol{x} * Out$ (where Out is the network output), and the following initial weights: $\boldsymbol{w} = (0.3, -0.2)$. Consider the following training examples:

x_1	x_2	y
–0.1	0.1	1
0.2	0.15	–1

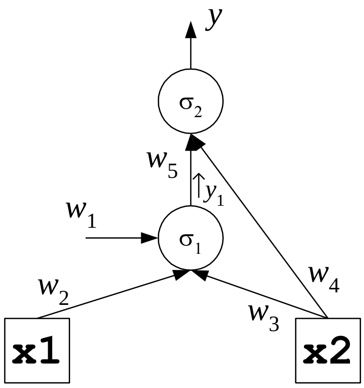
Show the computation of the weight corrections $\Delta\boldsymbol{w}$ after each example. **[10]**

Question 3.

a) i) Give the gradient descent training rule for multilayer networks with weight decay regularization and explain its components. **[4]**

ii) What is the effect of using regularization in training feed-forward multilayer networks? **[3]**

b) A multilayer neural network with two nodes: one hidden and one output using sigmoidal activations is given. There are two inputs to the network: (x_1, x_2), and 5 weights as illustrated in the picture below. Train this network using the backpropagation algorithm with learning rate 1.



Consider the following initial weights:

$$w_1 = -1.0 \quad w_2 = 0.2 \quad w_3 = 0.3 \quad w_4 = -0.5 \quad w_5 = -0.6$$

Use the following training vector:

x_1	x_2	y
1	1	0

Perform one iteration of the backpropagation algorithm, and show the output, the error derivatives β (beta), the weight updates, and the final modified weights. **[18]**

Question 4.

a) Give a definition of unsupervised learning and relate it to self-organized learning. **[3]**

b) Consider a self-organizing Kohonen network with 2 neurons each of which accepts 4 inputs.

i) Draw a picture of this one-dimensional self-organizing network showing the labeling of the weights and the neurons. **[4]**

ii) Define two alternative formulae for computing the distance between the input vector and the training example in the Euclidean space. **[6]**

iii) Determine the winning neuron using the alternative to the product vector formula when the following input vector is given: $\mathbf{x} = (\ 0.2,-0.1,0.3,1\ )$. Assume the following weight vectors: $\mathbf{w}_1 = (\ 1,-0.4,0.2,0.1\ )$ and $\mathbf{w}_2 = (\ 0.3,1,1,-0.2\ )$. **[8]**

iv) Update the weights of the winning neuron using learning rate $\eta=0.25$. **[4]**

Question 5.

a) i) Explain briefly how the energy of the Hopfield network changes when there is a state change during training. **[4]**

ii) Define the formula for computing the neuron output in Hopfield networks. **[2]**

b) Let a simple Hopfield recurrent network with 3 neurons and 3 inputs be given. Suppose that the initial weights matrix is:

$$\mathbf{W} = \begin{matrix} 0 & -0.3 & 0.2 \\ -0.3 & 0 & 0.2 \\ 0.2 & 0.2 & 0 \end{matrix}$$

i) Calculate the energy of this Hopfield network for the two states: [1,0,1] and [1,0,0]. **[6]**

ii) Show that the stable state for this network is: [1,0,1]. **[6]**

iii) Demonstrate that this Hopfield network is unstable for the third neuron given input vector [1,0,0], and retrain it. **[7]**