UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B.Sc. Examination 2012

Computing and Information Systems and Creative Computing

IS51002c (CIS102c)
Mathematical Modelling for Problem Solving

Duration: 2 hours 15 minutes

Date and time:

There are five questions in this paper. You should answer no more than THREE questions. Full marks will be awarded for complete answers to a total of THREE questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 75 marks available on this paper.

Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) The first 16 hexadecimal integers ≥ 0 can be represented by 4 binary strings as follows:

0000:0	0100:4	1000:8	1100:C
0001:1	0101:5	1001:9	1101:D
0010:2	0110:6	1010:A	1110:E
0011:3	0111:7	1011:B	1111:F

(1) Find the hexadecimal equivalent of the binary numeral 1101110.101.

(2) Find the binary of the hexadecimal numeral B09.A.

(3) Working in Hexadecimal system, compute the following sum. showing all your workings.

$$11001001 - 100111$$

[8]

(b) (1) Define what is meant by a rational number.

(2) Showing all your working, express the repeating decimal $0.272727\cdots$ as a fraction in its simplest terms.

[6]

(c) (1) Let $A = \{2, 4, 8, 16, \cdots, 1024\}$ and $B = \{3n - 1 : n \in \mathbb{Z}^{+}\}$.

(i) Describe the set A by the rule of inclusion method.

(ii) Describe the Set B by the listing method.

(2) Let A, B and C be three subsets of a universal set U.

(i) Draw a labeled Venn Diagram showing A, B and C intersecting in the most general way.

(ii) Shade the area for $X = A \cap (B' \cup C)$.

(iii) Show that: $X = (A' \cup (B \cap C'))'$.

[11]

Question 2

(a) Let $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer n in S

$$p \;:\; n \textit{ is a multiple of two}$$
$$q \;:\; n \textit{ is a multiple of three.}$$

(1) Find the set of values for which each of the following compound statements is true:

$$p \wedge q;\ p \vee q;\ p \wedge \neg q$$

(2) Express the following statement symbolically:

n is not a multiple of two or three.

(3) List the elements of the truth set for the statement in (2).

[8]

(b) (1) Let p and q be propositions. Use truth tables to prove that

$$p \rightarrow q \equiv \neg q \longrightarrow \neg p$$

(2) Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 0, then n is divisible by 5.

[7]

(c) Let the sequence u_n be defined by the following recurrence relation:

$$u_{n+1} = u_n + 2n, \quad \textit{for all } n \geq 1 \textit{ and } u_1 = 1$$

(1) Calculate u_2, u_3, u_4 and u_5, showing all your working.

(2) Prove by mathematical induction that

$$u_n = n^2 - n + 1, \quad \textit{for all } n \geq 1.$$

(3) Showing all your working, find the sum of the first 100 terms of this sequence.

[10]

Question 3

(a) Given any real number x ($x \in \mathbb{R}$), the floor value is denoted by $\lfloor x \rfloor$ and the absolute value is denoted by $|x|$.

(1) Find $\lfloor \sqrt{3} \rfloor$ and $|-3|$.

(2) Find the set of values of a such that $\lfloor a \rfloor = 2$ and the set of values of b such that $|b| = 2$.

(3) Consider the function $f : \mathbb{R} \to \mathbb{Z}$ and $g : \mathbb{R} \to \mathbb{R}$ is given by

$$f(x) = \lfloor x - 2 \rfloor \quad and \quad g(x) = |x - 2|$$

(i) Write down the domain, co-domain and range of f and g.

(ii) For each function, say whether or not it is one to one, justifying your answer.

(iii) For each function, say whether or not it is onto, justifying your answer.

[12]

(b) (1) State the condition to be satisfied in order for a function to have an inverse.

(2) Given the function $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 5x - 2$.

(i) Show that f is a one to one function.

(ii) Show that f is an onto function.

(iii) Find the inverse inverse function f^{-1}.

(3) Let g be a function defined as follows:
$g : \mathbb{Z} \to \mathbb{R}$ where $g(x) = 5x - 2$.

(i) Is g a one to one function? Explain your answer.

(ii) Is g an onto function? Explain your answer.

(iii) Is g invertible? Explain your answer.

[13]

Question 4

(a) What properties should a graph have in order for it to be:

(1) a simple graph;

(2) a complete graph;

(3) a tree.

[6]

(b) (1) Say, with reason, whether or not it is possible to construct a simple graph with degree sequence 5,3,2,2,2.

(2) Let K_n be a complete graph with n vertices, $v_1,\ v_2,\ v_3, \cdots, v_n$.

(i) Draw K_6.

(ii) Determine the number of edges of K_6.

(iii) Determine the number of paths from v_1 to v_2 of length two.

(iv) Find an expression in terms of n for the number of paths from v_1 to v_2 of length two in k_n.

[13]

(c) A binary search tree is designed to store an ordered list of 2000 records at its internal nodes.

(1) Find the records stored at the root (level 0) and level 1 of the tree.

(2) What is the height of the tree?

[6]

Question 5

(a) Given the set $S = \{\{a,b\},\{a\},\{b\},\{a,b,c\}\}$. $\mathcal{R}$ is a relation defined on S as follows:

$$X\ \mathcal{R}\ Y\ \textit{if}\ X \subseteq Y\ \textit{where}\ X, Y \in S$$

(1) Draw the relationship digraph $\mathcal{R}$.

(2) Say, with reason, whether $\mathcal{R}$ is reflexive, symmetric, anti-symmetric and transitive.

(3) State, with reason, whether $\mathcal{R}$ is a partial order or not.

(4) Is $\mathcal{R}$ a total order? Explain your answer.

[12]

(b) Let S be the set $\{5, 6, 7, 8, 9, 10\}$ and let $\mathcal{R}$ be a relation defined between the elements of S by

$$x \text{ is related to } y \text{ if } \ (x - y)\ \textit{mod}\ 2 = 0.$$

(1) Draw the relationship digraph for $\mathcal{R}$.

(2) Determine whether or not $\mathcal{R}$ is reflexive, symmetric or transitive. In cases where one of these properties does not hold give an example to show that it does not hold.

(3) State, with reason, whether $\mathcal{R}$ is a partial order or not.

(4) Is $\mathcal{R}$ an equivalence relation? If yes find the equivalence classes.

[13]