UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2012

All Computing Programmes

IS51002B (CIS102B) Mathematics for Computing

Duration: 3 hours

Date and time:

There are TEN questions on this paper.

Full marks will be awarded for complete answers to TEN questions.

Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1 Numbers

(a) Working in base 2 and showing all your working, compute the following: [4]

i. $(10101)_2 + (1111)_2$

ii. $(11011)_2 - (11)_2$

iii. $(10101)_2 \times (11)_2$

(b) Express the binary number $(1011.01)_2$ as a decimal, showing all your working. [2]

(c) Express the decimal number $(349)_{10}$ in base 2. [2]

(d) Say to which of the sets $\mathbb{N}$, $\mathbb{Q}$ or $\mathbb{R}$ the following numbers belong. If they belong to more than one of these sets give all the sets. [2]

i. π

ii. $\frac{2}{3} + 5$

Question 2 Sets

(a) i. Describe the following set by the rule of inclusion method:
{the set of integers which have a remainder of 1 on division by 4.}

ii. Describe the following set by the listing method:
{the set of positive multiples of 5 which are less than 80.}

[3]

(b) Let A, B and C be subsets of a universal set $\mathcal{U}$. Shade the following regions on a Venn diagram:

[5]

$A \cap B \cap C'$

$(A \cup B') \cap C$

(c) You are given the following expressions:

$\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8, 9\}$

$B = \{1, 2, 3, 4, 5, 6, 7\}$

List the elements in the following set [2]

$(A' \cup B)'$

Question 3 Logic

(a) Let $n \in \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$ and let p, q be the following propositions concerning the integer n.

$$p\colon n \text{ is a multiple of two}$$
$$q\colon n \text{ is a multiple of three.}$$

i. Find the set of values of n for which each of the following compound statements is true: [4]

$p \wedge q$

$p \vee q$

$\neg p \oplus q$

(b) List the elements of the truth set for the statement $p \vee q$. [2]

(c) Let p and q be propositions. Use truth tables to demonstrate under what interpretations the following expression is true or false:

[4]

$(p \to \neg q) \to (q \to \neg p)$

Question 4 Relations

(a) Suppose we have the following set.

$S = \{1, 2, 3, 4, 5, 6\}$

Also suppose we have a relation R defined on S by the following condition.

$xRy \Leftrightarrow (x + y) \bmod 3 = 0$

In other words two elements are related if they have the same remainder when divided by 3.

i. Draw the digraph of R.

ii. Say with reason whether or not R is

- reflexive
- symmetric
- transitive

In the cases where the given property does not hold provide a counter-example to justify this.

[5]

(b) Another relation is defined on S in the following way.

$xRy \Leftrightarrow x \bmod 3 = y \bmod 3$

In other words two elements x and y are related if their sum is divisible by 3

i. Draw the digraph of this relation on S.

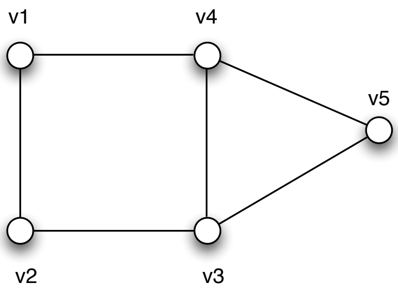
ii. Is this relation an equivalence relation or a partial order? Justify your answer.

iii. List the set of equivalence classes.

[5]

Question 5 Matrices and Graphs

(a) Consider the following graph G. [5]

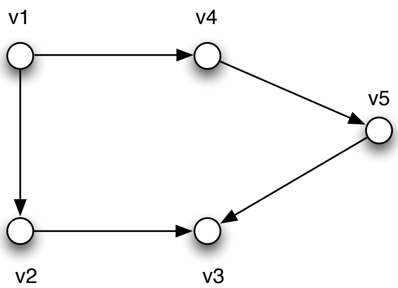


i. Write down the adjacency matrix for this graph A.

ii. Compute A^2 and thus state the number of walks of length 2 from:

$v1$ to $v2$
$v2$ to $v3$
$v2$ to $v2$

(b) Consider the following directed graph G. [5]



i. Write down the adjacency matrix for this graph B.

ii. Compute B^2 and thus state the directed walks of length 2.

iii. Calculate B^3 and thus state the number of walks of length 3.

Question 6 Matrices and Transformations in 2D

(a) The transformation matrix for the anti-clockwise rotation of 90 degrees is given below.

$$A = \begin{bmatrix} 0 -1 \\ 1 \;\; 0 \end{bmatrix}$$

Use this matrix to calculate the transformation matrices for 180, 270 and 360 degrees. [3]

(b) Consider the following four points below that form a rectangle in Euclidean space.

$(0, 0), (3, 0), (3, 1), (0, 1)$

Sketch what happens to this triangle when the following transformation is applied. [4]

$$A = \begin{bmatrix} 0 -1 \\ 1 \;\; 0 \end{bmatrix}$$

Hence or otherwise find

$$\begin{bmatrix} 0 -1 \\ 1 \;\; 0 \end{bmatrix}^{-1}$$

(c) Consider what happens to the rectangle described above when the following transformation is applied to the same rectangle and determine whether or not it has an inverse. [3]

$$A = \begin{bmatrix} 1 & 5 \\ 1 & 5 \end{bmatrix}$$

Question 7 Matrices and Homogenous Coordinates

(a) Describe what is meant by Homogeneous Coordinates. [2]

(b) State with explanation whether the three points listed below are homogenous coordinates for (3,4) and why. [1]

$(12, 16, 4)$

$(15, 20, 5)$

$(300, 400, 100)$

(c) Write down the general form of a point on the plane $z = 1$. [1]

(d) Apply each of the following matrices to a general point on the $z = 1$ plane and use this to determine the nature of the transformation it represents. [6]

i.

$$A = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

ii.

$$A = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iii.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 8 Sequences and Series

The sum of the first n terms of an arithmetic progression with first term a and common difference d is given by the following formula.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

The sum of the first n terms of a geometric progression with first term a and common ratio r is given by the following formula.

$$S_n = \frac{a(r^n - 1)}{(r-1)}$$

Consider the following sequences.

(i) $1, 3, 5, 7, \cdots$

(ii) $3, 5, 9, 17, \cdots$

(iii) $1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \cdots$

(iv) $1, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots$

(a) For each sequence write down the next two terms. [2]

(b) Identify each sequence as arithmetic, geometric or neither. If you identify it as arithmetic, specify the common difference d. If you identify it as geometric, specify the common ratio r. [2]

(c) Write down the nth term (u_n) for each sequence. [2]

(d) Calculate the sum of the first 10 terms of the first sequence. [2]

(e) From the formula above deduce the formula for the infinite sum of a converging series. Then, for any convergent series in the list above calculate the sum to infinity. [2]

Question 9 This question is about sketching the shape of functions

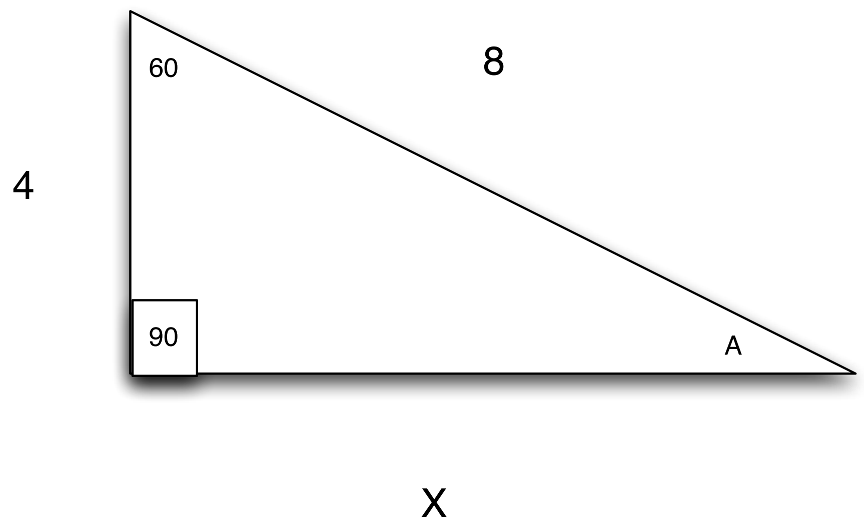
(a) Sketch the following graphs showing where the graphs cross the x-axis and y-axis. [5]

i. $y = x^2$
ii. $y = (x^2 - 1)$
iii. $y = (x)^2$
iv. $y = (x - 1)^2(x - 1)$
v. $y = -(x - 1)(x - 2)(x - 3)(x - 2)$

(b) Decide whether the following functions are odd, even, periodic or otherwise explaining your answer. [5]

i. $f(x) = x^2 - 4$
ii. $f(x) = x^3 - 5x$
iii. $f(x) = sin^2(x) + cos^2(x)$
iv. $f(x) = tan(x) - 1$
v. $f(x) = sin(x + \pi)$

Question 10 This question is about trigonometry



Please consider the diagram above.

(a) What is the value of A? [1]

(b) What is the value of X? [1]

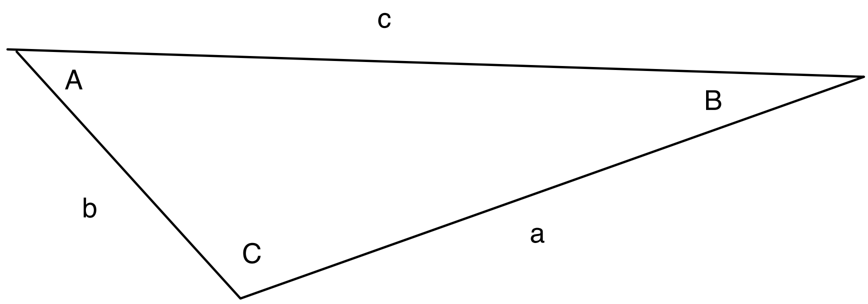
(c) What is $sinA$ leaving your answer as a fraction? [1]

(d) What is $cosA$ leaving your answer as a fraction? [1]

(e) What us $tanA$ leaving your answer as a fraction? [1]

(f) Demonstrate that in this case $sin^2 A + cos^2 = 1$. [2]

The cosine rule is defined in the next diagram.



Cosine Rule

$$a^2 = b^2 + c^2 - 2*b*c*\cos A$$

You are also told that one boat sets off sailing from a port Due East at a constant speed of 4 miles an hour at 3pm. An hour later another boat sets sail at 5 miles an hour in the North West direction. Using the cosine rule or otherwise determine how far away the boats are at 6pm leaving your answer as a number and a quotient. (You do not need to calculate this number). You may need to use the result [3]

$cos\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$