UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B.Sc. Examination 2011

Computing and Information Systems and Creative Computing

IS51002c CIS102c Mathematical Modelling for Problem Solving

Duration: 90 minutes

Date and time:

There are four questions in this paper. You should attempt all questions. Full marks will be awarded for complete answers to four questions. Each question carries 25 marks. The marks for each part of a question are indicated at the end of the part in [.] *brackets.*

There are 100 marks available on this paper.

Electronic calculators may not be used.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) Convert the decimal integer $(507)_{10}$ to binary notation. [4]

(b) Working in binary and showing all carries, compute $(11010)_2 + (111)_2$. [3]

(c) i. List the set of positive integers with precisely 3 bits in binary notation.

ii. Let n be a positive integer. How many positive integers have precisely n bits in binary notation? [6]

(d) Let the two sets A and B be given by

$$A = \{3, \tfrac{1}{3}, \pi\} \qquad \text{and} \qquad B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}.$$

Give each of the following sets by using the listing method.

i. $A \cap \mathbb{Q}$;

ii. $A \cap \mathbb{R}$;

iii. $A \cap B$;

iv. $A - B$. [8]

(e) Describe by using the rules of inclusion method the set of non-negative integers which have a remainder of 0 on division by 10. [4]

Question 2

Let p, q and r be the following propositions concerning integers n.

$$\begin{aligned} p &: \quad n \text{ is a multiple of two} \\ q &: \quad n < 20 \\ r &: \quad n \leq 20. \end{aligned}$$

(a) List the truth set of the compound proposition $\neg q \wedge p$. [3]

(b) Express each of the following statements using the propositions p, q and r and logical symbols.

i. n is an integer less than 20 which is even;

ii. n is an integer larger than 20 which is odd;

iii. $n = 20$. [6]

(c) Use truth tables to prove that

$$p \rightarrow (q \vee r) \equiv (\neg q \wedge \neg r) \rightarrow \neg p.$$

[5]

(d) A sequence is given by the recurrence relation

$$u_n = u_{n-1} + 2n \qquad \text{for } n \geq 2$$

and the initial term $u_1 = 1$.

i. Showing your working, calculate u_2, u_3, u_4 and u_5. [4]

ii. It can be proven by induction that $u_n = n^2 + n - 1$ for all $n \geq 1$. Showing all your working, compute

$$\sum_{n=1}^{100} (u_n - (n-1)^2).$$

[7]

Question 3

(a) Given a real number x, say how $\lfloor x \rfloor$, the *floor* of x, is defined. [2]

(b) The function $f : \mathbb{R} \to \mathbb{R}$ is given by the rule

$$f(x) = \lfloor x/2 \rfloor.$$

i. Find $f(-3)$ and $f(3)$.

ii. Justifying your answer, say whether f is one-to-one.

iii. Justifying your answer, say whether f is onto. [10]

(c) Let S be the set $\{5, 6, 7, 8, 9, 10\}$ and a relation $\mathcal{R}$ is defined between the elements of S by

$$x \text{ is related to } y \text{ if } \ (x - y) \in \ \{0,2,4\}.$$

i. Draw the relationship digraph.

ii. Determine whether or not $\mathcal{R}$ is reflexive, symmetric or transitive. In cases where one of these properties does not hold give an example to show that it does not hold.

iii. State, with reason, whether $\mathcal{R}$ is a partial order or not.

[13]

Question 4

(a) i. Let G be a simple graph. How is the sum of the degrees of the vertices of G related to the number of edges of G? [3]

ii. Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$3, 1, 1, 1, 1.$$

[3]

iii. Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$5, 4, 3, 2, 1.$$

[3]

iv. Justifying your answer, say whether it is possible to construct a simple graph with precisely 50 vertices and 1500 edges. [4]

(b) i. Construct a binary search tree to store the following ordered list of 12 integers at its internal nodes.

$$1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23.$$

ii. What is the maximum number of comparisons needed in order to find an existing integer in the tree? [8]

(c) A binary search tree is designed to store an ordered list of 300 records at its internal nodes.

i. Which record is stored at the root (at level 0) of the tree?

ii. Which records are stored at level 1 of the tree? [4]