UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Foundation Year Examination 2011

Computing

IS50002A (FY02)   Foundation Mathematics

Duration: 3 hours

Date and time:

There are ten questions in this paper. You should attempt all questions. Full marks will be awarded for complete answers to ten questions. Each question carries 10 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Electronic calculators may not be used.

THIS PAPER MUST NOT BE REMOVED
FROM THE EXAMINATION ROOM

Question 1   This question is about numbers, fractions and the binary and hexadecimal number systems.

(a) Convert the following:

    i. The fraction $\frac{7}{8}$ into a decimal.

    ii. The binary number 1011 into a decimal number.

    iii. The decimal 0.28 into a fraction in its lowest terms.

    iv. The decimal number 45 into a binary number.

[4]

(b) Calculate the following, writing your answers as fractions in their simplest form.

    i. $\frac{5}{12} - \frac{3}{8}$

    ii. $3\frac{1}{3} \div \frac{8}{9}$.

[2]

(c) Calculate the following and show all your working in binary.

    i. $1010_2 + 1111_2$

    ii. $10101_2 - 1011_2$.   [2]

(d) Convert the following numbers into hexadecimal.

    i. 211 in base ten

    ii. 1101010 in base two.

[2]

Question 2   This question is about ratios, scientific notation, decimal places and significant figures.

(a) A blend of muesli contains oats, raisins and nuts in the ratio $5 : 3 : 2$.
Calculate how many grams of each there are in a 400g box of muesli.

[2]

(b) i. 2.34 is a number written correct to two decimal places. Write down the smallest number it could be.

    ii. Write 0.0963 correct to two decimal places.

    iii. Which of the following is equivalent 5.5 when they are corrected to one decimal place: 5.497 or 5.597?

[3]

(c) Write the following as decimals and then correct them to 2 significant figures.

    i. 32.4%

    ii. 1.25%

    iii. 108.75%.

                [3]

(d) Express the following numbers in scientific notation.

    i. 434343

    ii. 0.00005678.

                [2]

Question 3   **This question is about sets, logic and Venn diagrams.**

(a) Complete truth tables for the following and hence decide whether it is a contingency, tautology or a contradiction.

$$(p \wedge \neg q) \vee \neg p$$

                [3]

(b) The sets A, B and C are subsets of a universal set U as follows.

$$U = \{1, 2, 3, ..., 20\}$$

$$\text{A} = \{\text{multiples of two}\}$$
$$\text{B} = \{\text{ multiples of three}\}$$
$$\text{C} = \{\text{ multiples of five}\}.$$

List the following sets:

    i. $A \cap B$

    ii. $B \cap C$

    iii. $(A \cup B) \cap C$

    iv. $A \cap \overline{C}$

    v. $\overline{A \cup C}$.

                [5]

(c) Shade the following regions on separate Venn diagrams for general sets P and Q.

    i. $P \cap \overline{Q}$

    ii. $\overline{P \cup Q}$.

                [2]

Question 4   **This question is about Series.**

Consider the following series.

i. $\sum_{k=1}^{100} (2k - 1)$

ii. $\sum_{k=1}^{20} (4 - 5k)$

iii. $\sum_{k=1}^{20} 3^k$

iv. $\sum_{k=1}^{4} \frac{2k}{k+1}$.

(a) Write down the first three terms of each series.   [4]

(b) Decide which of the 4 series are arithmetic, geometric or neither.   [2]

(c) Find the sum of series i) using the appropriate formula.   [2]

(d) Find the sum of series iv) leaving your answer as a fraction.   [2]

Question 5   **This question is about linear, simultaneous and quadratic equations.**

(a) Solve the following equations.

   i. $4(3x - 1) = 16$
   ii. $\frac{x-4}{3} + 2x = 8$.

                [2]

(b) Solve the following quadratic equations.

   i. $x^2 - 5x - 14 = 0$ by factorisation
   ii. $x^2 - 6x + 8 = 0$ by factorisation
   iii. $2x^2 - 3x - 9 = 0$ using the formula.

                [5]

(c) Solve the following simultaneous equations.

$$x - 2y = -4$$

$$5x + y = -9.$$

                [3]

Question 6   This question is about functions.

Consider the following functions.

$$f(x) = x + 3; \quad g(x) = \tfrac{1}{2}x; \quad h(x) = x^2.$$

(a) Evaluate the following.

   i. $f(-5)$

   ii. $g(f(-1))$

   iii. $g(f(h(3)))$.

                                                                  [3]

(b) Write an expression for $f(2x + 1)$ and $h(2x + 1)$.   [2]

(c) Find $f^{-1}(x)$ and $g^{-1}(x)$.   [3]

(d) Write an expression for $f(g(h(x)))$.   [2]

Question 7   This question is about plotting graphs.

(a) Consider the following equations.

$$y = x - 3, \quad y = -3x + 2.$$

Plot the straight lines represented by these equations on graph paper for $-3 \le x \le 3$. Find the point of intersection of these lines.   [4]

(b) The graph of $y = x^2 - bx - 16$ cuts the x axis at $x = -2$ and $x = 8$ and the y axis at -16. Sketch this graph for $-3 \le x \le 9$ and hence find the value of b and find the co-ordinates of its vertex.   [4]

(c) Sketch the following graphs, showing where they cut the axes.

   i. $y = 2^x$

   ii. $y = 2^x - 3$.

                                                                  [2]

Question 8   This question is about trigonometry.

(a) Sketch a right angled triangle with base 6 and height 8 units. The hypoteneuse of this triangle is h units.

    i. Find h.

    ii. Let A be the angle between the base and the hypoteneuse. Find the following, leaving your answer as a fraction.

$$\cos A; \quad \sin A.$$

[5]

(b) i. Given that $\sin 60° \approx 0.866$ and $\sin 30° = 0.5$, use graph paper to sketch the graph of $y = \sin x$ for $-360° \le x \le 360°$, showing where it cuts the x-axis.

    ii. Use your graph to find any other possible angles whose sine is equal to 0.5 which lie between -360 and 360 degrees.

[5]

Question 9   This question is about matrices.

(a) Evaluate the following.

    i.

$$-2 \begin{pmatrix} -3 & 2 \\ 4 & -2 \end{pmatrix} - \begin{pmatrix} 7 & -5 \\ -1 & -3 \end{pmatrix}$$

    ii.

$$\begin{pmatrix} 2 & 4 \\ 6 & 3 \end{pmatrix} \begin{pmatrix} 7 & 9 \\ 1 & 8 \end{pmatrix}$$

[4]

(b) i. Write down the 2 by 2 identity matrix, $\mathbf{I}$.

    ii. Find the inverse of the following matrix $\mathbf{M}$.

$$\begin{pmatrix} -5 & 8 \\ 2 & -3 \end{pmatrix}$$

    iii. What is the result of pre-multiplying this inverse by the identity matrix $\mathbf{I}$?

    iv. What is the result of pre-multiplying this inverse by the original matrix $\mathbf{M}$?   [6]

Question 10   **This question is about probability.**

(a) This is about five digit codes such as 45544 or 29036, using the digits from 0,1,2,3,...9, where each digit may be repeated up to five times.

    i. How many different codes are there with five digits?

    ii. How many different codes are there starting with a 9?

    iii. How many different codes are there starting and ending with a 9?

[3]

(b) I throw a fair coin 3 times.

    i. What is the probability I get exactly two tails?

    ii. What is the probability I get at least two tails?

[2]

(c) A draw contains 9 socks of which 3 are red and 6 are green. I now pick out two socks, one after the other at random without replacement.

    i. Draw a tree diagram to show the possible outcomes of this process.

    ii. Find the probability that both socks are red.

    iii. Find the probability that the socks are different colours.

[5]