UNIVERSITY OF LONDON

EXTERNAL PROGRAM

B. Sc. Examination 2007

COMPUTING

CIS102w Mathematics for Computing

Duration: 3 hours

Date and time:

There are TEN questions on this paper.
Full marks will be awarded for complete answers to TEN questions.
Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) The first 16 integers ≥ 0 can be represented by 4 bit binary strings.

(i) List these integers in hexadecimal, together with their binary equivalents.

(ii) Find the hexadecimal equivalent of the binary numeral 100101.01 and find the binary equivalent of the hexadecimal numeral $59.A$ [4]

(b) Working in the binary system compute the following sum, showing all your working:

$$(110111)_2 + (1010111)_2 + (1110111)_2.$$

[2]

(c) (i) Define what is meant by an irrational number. Say whether or not the repeating decimal 0.17321732..... is a rational or irrational number, justifying your answer.

(ii) Showing all your working, express the repeating decimal 0.270270..... as a fraction in its simplest terms. [4]

Question 2

(a) Let $A = \{2n : n \in \mathbb{Z}^+\}$ and $B = \{3, 6, 9, 12, ...\}$ be two sets of numbers.

(i) Describe the set A by the listing method.

(ii) Describe the set B by the rules of inclusion method.

(iii) Find the two sets $A \cap B$ and $A - B$, by the listing method. [5]

(b) Let P, Q and R be subsets of a universal set $\mathcal{U}$.

(i) Construct a membership table for the set $X = P' \cup (Q \cap R)$.

(ii) Draw a labelled Venn diagram showing P, Q, and R intersecting in the most general way.

(iii) Shade the region X on your diagram.

(iv) Is the set $Q \cap R \subseteq X$? Justify your answer. [5]

Question 3 (a) Let n be a positive integer and p and q be the following propositions:

$$\begin{aligned} p &: n \leq 12 \\ q &: nisodd. \end{aligned}$$

(i) Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

$$\begin{aligned} n &\leq 12andniseven. \\ ifn &\leq 12thenniseven \\ n &> 12andnisodd. \end{aligned}$$

(ii) Construct the truth table for the statement $q \rightarrow p$. Hence find a value of n that makes this statement false.

(iii) Write in logical symbols the contrapositive of the statement:

$$ifnisoddthenn \leq 12.$$

[6]

(b) Construct a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output

$$\neg(\neg p \wedge q).$$

Show the truth table for this output and hence give a simple expression (without using negation) that is equivalent to $\neg(\neg p \wedge q)$. [4]

Question 4

(a) Given $u_k = 5k + 1$ and $s_n = \sum_{k=1}^{n} (5k + 1)$ for all positive integers n.

(i) Calculate $u_1,\ u_2,\ u_3$ and u_4.

(ii) Calculate $s_1\ s_2$ and s_3.

(iii) Use the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ to find a formula for $s_n = \sum_{k=1}^{n} (5k + 1)$ in terms of n. Use this formula to find this sum when $n = 10$. [6]

(b) Prove by induction that

$$3 + 7 + 11 + 15 + ... + (4n - 1) = n(2n + 1) forallpositiveintegersn.$$

[4]

Question 5

(a) There are 16 different 2 by 2 matrices whose entries may consist only of zeroes and ones, for example

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} and \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} aretwosuchmatrices.$$

Let S be the set all such matrices. We define a function f on S by the rule

$$f(\mathbf{X}) = thenumberofzeroesin\mathbf{X}wheref : S \to \mathbb{Z}and\mathbf{X} \in S.$$

(i) Find a numerical value for both $f(\mathbf{A})$ and $f(\mathbf{B})$.

(ii) Write down the set of pre-images or ancestors of 1.

(iii) Write down the range of f.

(iv) Say whether or not this function is one to one, justifying your answer.

(v) Say whether or not this function is onto, justifying your answer. [6]

(b) Say whether or not each of the following functions has an inverse, justifying your answer. In the cases where there is an inverse define it.

(i) $f : S \to \mathbb{Z}$ defined in part (a).

(ii) $g : \mathbb{R} \to \mathbb{Z}$ defined by $g(x) = \lfloor x \rfloor$.

(iii) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 2x + 5$. [4]

Question 6 Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, ..., v_n$ and edges $\{v_1,v_2\}, \{v_2,v_3\}, ...\{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex, v_{n+1}, and edges from this to each of the original vertices in C_n.

(a) Draw K_4, C_4, and W_4. [$2\frac{1}{2}$]

(b) Giving your answer in terms of n, write down an expression for the number of edges in K_n, C_n, and W_n. [$2\frac{1}{2}$]

(c) (i) Find the number of different paths of length two in each of the graphs in part (a), where a path does not contain the same edge more than once, and a path from v_x to v_y is different from a path from v_y to v_x.

(ii) Giving your answer in terms of n, write down an expression for the number of different paths of length two there are in K_n. [5]

Question 7 Given a flock of chickens, between any two chickens one of them is dominant. A relation, R, is defined between chicken x and chicken y as xRy if x is dominant over y. This gives what is known as a pecking order to the flock. Home Farm has 5 chickens: Amy, Beth, Carol, Daisy and Eve, with the following relations:

$$Amy is dominant over Beth and Carol$$
$$Beth is dominant over Eve and Carol$$
$$Carol is dominant over Eve and Daisy$$
$$Daisy is dominant over Eve, Amy and Beth$$
$$Eve is dominant over Amy.$$

(a) Draw a digraph to represent this pecking order, saying what the vertices represent and what it means when two vertices are connected by an edge. [2]

(b) Say whether or not the pecking order R is

- (i) reflexive;
- (ii) anti-symmetric;
- (iii) transitive;
- (iv) a partial order.

  Justify each answer in terms of a small proof or counter-example. [4]

(c) Another relation, R_2, is defined between the chickens on Home Farm. Let x and y be chickens on Home Farm, then

$$xR_2y if and only if x and y have the same mother.$$

The mothers of the chickens on Home Farm are either Flora or Harriet from a neighbouring farm. Harriet is the mother of Amy, Daisy and Eve. Flora is the mother of Beth and Carol.

Justifying your answer, say whether R_2 is an equivalence relation on the set of chickens at Home Farm. If this is an equivalence relation write down the equivalence classes. [4]

Question 8 Given S is the set of all 5 digit binary strings, E is the set of a 5 digit binary strings beginning with a 1 and F is the set of all 5 digit binary strings ending with two zeroes.

(a) Find the cardinality of S, E and F. [3]

(b) Draw a Venn diagram to show the relationship between the sets S, E and F. Show the relevant number of elements in each region of your diagram. [3]

(c) What is the probability that a 5 digit binary string chosen at random :

(i) begins with a 1;

(ii) ends with two zeroes;

(iii) both begins with a 1 and ends with two zeroes;

(iv) either begins with a 1 or ends with two zeroes or both? [3]

(d) Say whether or not E and F are independent events, justifying your answer. [1]

Question 9

(a) Given the graph G with vertices $v_1,\ v_2, \ldots\ v_7$ and adjacency list

$v_1 : v_2, v_4$

$v_2 : v_1, v_3$

$v_3 : v_2, v_4$

$v_4 : v_1, v_3, v_5$

$v_5 : v_4, v_6$

$v_6 : v_5, v_7$

$v_7 : v_5, v_6$.

(i) Draw this graph.

(ii) Say how many edges there are in a tree with n vertices. Hence explain how many edges must be removed from G to create a spanning tree.

(iii) The graph G has precisely 12 different spanning trees, list the twelve distinct pairs of edges which, when removed, give the 12 spanning trees, $T_1,\ T_2, \ldots.\ T_{12}$.

(iv) By partitioning the set $\{T_1,\ T_2, \ldots.\ T_{12}\}$ into subsets where the trees of a subset are all isomorphic to one another, while the two trees from different subsets are non-isomorphic, or otherwise, draw the four non-isomorphic spanning trees of G. [7]

(b) A binary search tree is designed to store an ordered list of 50000 records, numbered 1,2,3....50000 at its internal nodes.

(i) Draw levels 0, 1 and 2 of this tree, showing which number record is stored at the root and at each of the nodes at level 1 and 2, making it clear which records are at each level.

(ii) What is the maximum number of comparisons that would have to be made in order to locate an existing record from the list of 50000? [3]

Question 10

(a) Given the following adjacency matrices **A** and **B** where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}.$$

(i) Say whether or not the graphs they represent are isomorphic.

(ii) Calculate $\mathbf{A}^2$ and $\mathbf{A}^4$ and say what information each gives about the graph corresponding to **A.** [6]

(b) (i) Write down the augmented matrix for the following system of equations.

$$\begin{aligned} 2x + y - z &= 2 \\ x - y + z &= 4 \\ x + 2y + 2z &= 10 \end{aligned}$$

(ii) Use Gaussian elimination to solve the system. [4]

END OF EXAMINATION