UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2003

MATHEMATICS

IS53016A(CIS331) Introduction to Mathematical Modelling in Management Science

Duration: 2hours 15minutes

Date and time:

There are six questions on this examination paper.
Do not attempt more than FOUR questions.
Full marks will be awarded for complete answers to FOUR questions.

Electronic calculators may be used. The make and model should be specified on the script. The calculator must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Candidates may use GRAPH PAPER, which will be provided.

BEGIN EACH QUESTION ON A NEW PAGE and number the question and parts.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

(a) Manufacturing company NewThing have designed a combination television-kettle that boils automatically whenever the adverts come on. They have called it the AdBoil and are planning a sales strategy. They believe that demand is a function of price and estimate that if the price is £x, then the monthly demand will be $7000e^{-0.005x}$ units.

At present, each unit they sell generates a cost of £182 and they have fixed costs of £24,000 per month. They build a spreadsheet model (as shown in cells A1:E7 of the attached spreadsheet) in such a way as to automatically calculate the demand and the profit, even if the price, cost-rate per unit and fixed monthly costs change.

NOTE: There ARE values missing from the spreadsheet. You have to calculate some of them below.

(i) State the Excel formulae used to calculate the values in cells B4, B5 and E4. [3]

(ii) Using the value for price shown in the spreadsheet (that is £300) calculate the values missing from B4, B5 and E4 and hence calculate the value of profit missing from cell B7. [5]

(b) Over the course of their first year selling the AdBoil, NewThing vary the price and monitor the demand as a result. They record one year's worth of figures and then use Excel to fit an Exponential curve through the data. This is shown in the attached spreadsheet. Also shown are the predictions made by NewThing's own estimate for the demand function and those made by the Exponential curve suggested by Excel.

To determine the accuracy of the original estimated demand function, Average Percentage Errors have been calculated for both sets of predictions.

(i) Calculate the value missing from cell E34. [3]

(ii) Calculate the Mean Average Percentage Error for both sets of predictions (that is the values missing from cells E44 and F44) and say why the curve fitted by Excel is the better model of how demand behaves with respect to price. [5]

(c) To start their second year, NewThing decide to set the price of the AdBoil somewhere between £500 and £700. They decide to keep using the Excel exponential curve to model demand, their fixed costs are unchanged, but due to a new accountancy structure, the variable cost associated with selling each unit is now (£75) + (26 percent of the selling price).

(i) Explain why NewThing should think very carefully before continuing to use the Exponential curve generated by the data from last year. [2]

(ii) If they fix the AdBoil price at £620, using the Excel Exponential model calculate the profit they will make in the first month of their second year. Show all your working. [7]

End of Question 1.

Question 2

A local newspaper is starting a regular feature that reviews recent rental film releases. In order to do this they wish to purchase a home cinema system for the journalists to use to view the movies. They already have a large television. They need to buy a DVD player, an amplifier and a speaker-package and they have £1000 to spend. Two local retailers each offer them a system set up that meets their budget and the newspaper must choose which to buy.

They produce the following pairwise comparison matrix to rate how important each of the three components are.

	DVD	Amp	Speaker
DVD	1	2	7
Amp	1/2	1	1/5
Speaker	1/7	5	1

However one of the reporters points out that this matrix is inconsistent. So they rethink and produce a new comparison matrix. After checking that the new matrix is consistent, they run the Analytical Hierarchy Process to determine which system to buy. The details (including the new pairwise comparison matrix) are presented on the attached spreadsheet.

NOTE: The attached spreadsheet DOES have some figures omitted. In parts of this question you will be asked to calculate the missing values.

(a) Explain (without using an algebraic test) why the first pairwise comparison matrix is inconsistent. [4]

(b) **Without refering to Excel functions,** describe carefully how to normalise a matrix. Illustrate your description by calculating the value missing from cell H6. Using this value, verify that the weight in cell J6 is correct. [5]

(c) Calculate the CI value in this situation and hence, given that the RI value for a three variable PCM is 0.58, determine whether or not the pairwise comparison matrix in cells B5:D7 is constistent, giving a reason for your answer. [5]

(d) Calculate the final scores for the machines (that is, the values that should occupy cells F23:F24). Thus state which system the newspaper should buy, giving a reason for your answer. [6]

(e) Arriving late at the meeting, one reporter looked at the weights vector in cells J5:J7 and suggested that they build their own system spending £591 on the DVD player, £334 on the amplifier and £75 on the speaker-package. Comment on two modelling issues that should be considered whilst deciding whether or not this a good idea. [5]

End of Question 2.

Question 3

Small games company Abacus makes a board game called AddEmUp, in both Regular and Travel editions. The Regular edition retails for £18 and the Travel for £12. Abacus buy the games as blank boards and generic chips. All they have to do is print the boards and package the games for sale.

Abacus employ one woman to do the printing. She works for 60 hours a week and can print a Regular board in 2 hours and the more fiddly Travel board in 3 hours. They have someone who works for 8 hours each week to do the packaging. He can package each Regular edition in 20 minutes and each Travel edition in 10 minutes. Also, Abacus can only obtain enough materials to make 15 Travel editions each week.

You have been asked to determine how many Regular and Travel editions Abacus should make each week in order to maximize their income. You are told to assume that they can sell as many games as they can make.

(a) Formulate this situation as a linear programming problem stating clearly the objective function, constraints and what the decision variables you use represent. [5]

(b) Using the graph paper provided, draw a graphical representation of the problem. Label each constraint line and clearly indicate the feasible region. [5]

(c) By plotting at least two suitable objective lines, locate the optimal point, calculate its position and hence state clearly how many Regular and Travel editions should be produced and the value of income achieved at this production level. [5]

(d) If the price of the Regular edition stays fixed at £18, find a value for the price of a Travel edition for which there will be infinitely many optimal points? [3]

(e) Describe briefly the nature of the assumptions of *proportionality, additivity* and *certainty* for linear programming models and comment on whether or not they are valid in this situation. [7]

End of Question 3.

Question 4

(a) Suppose we give the vertices of a network the labels $v_1, v_2, \ldots, v_n$. When can we say the labels form an acyclic ordering? [3]

(b) Consider the following precedence table for a multi-activity project. The durations given are all in days.

Activity	Depends on	Duration
A	-	2
B	-	6
C	A	3
D	A,B	5
E	D	7
F	D	6
G	C,E	4
H	G	6
I	G,F	6
J	F	3

(i) Draw an activity network to represent the project. Use as few dummies as possible and give the vertices an acyclic ordering. [8]

(ii) Find early and late times for the events in the project and indicate them clearly on your activity network. State the minimum project time. [5]

(iii) Explain carefully why there are two possible critical paths and state them both as lists of activities. [4]

(iv) It is discovered that activity D actually requires 8 days of work, but ativities E and F can still begin after 5 days of work on activity D. Describe how you would modify your activity network to show this information. Also, by checking appropriate early and late times, determine whether or not this new information changes the minimum project time. [5]

End of Question 4.

Question 5

(a) A communications engineer is designing an information network that consists of six nodes $v_1, v_2, \ldots, v_6$ and cables joining them. The cable is non-directional and costs £2 per metre. The distance between the nodes in metres is shown in the adjacency table below. The gaps indicate that it is impossible to connect two nodes directly.

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	-	14	20	-	-	-
v_2	14	-	12	15	19	-
v_3	20	12	-	34	-	42
v_4	-	15	34	-	9	16
v_5	-	19	-	9	-	20
v_6	-	-	42	16	20	-

By copying out the table and running Prim's algorithm, starting with vertex v_1, determine the minimum length of cable required to connect all the nodes. Finish by drawing a diagram showing how to connect the nodes with this minimum length of cable. [8]

(b) Because the network is used to transmit information from v_1 to the other nodes, another engineer decides to use a directional cable. She redesigns the table so that every edge has a direction. The adjacency table for her network is below.

		TO					
		v_1	v_2	v_3	v_4	v_5	v_6
FROM	v_1	-	14	20	-	-	-
	v_2	-	-	12	15	19	-
	v_3	-	-	-	34	-	42
	v_4	-	-	-	-	9	16
	v_5	-	-	-	-	-	20
	v_6	-	-	-	-	-	-

Draw the network represented by this table. [4]

(c) Use the shortest path algorithm to grow a tree that shows the shortest path from v_1 to each of the other nodes. Finish by drawing the tree you generate. [8]

(d) The cost of connecting the nodes using directional cable and the tree grown in part (c) is £27 less than the cost of using the non-directional cable and the tree from part (a). Showing all your working, calculate the cost per metre of the directional cable. [5]

End of Question 5.

Question 6

(a) Consider the following linear programming problem, P.
Find $x_1, x_2, x_3 \in \mathbf{R}$ to maximize $z = 4x_1 + x_2 + 7x_3$, subject to

$$\begin{aligned} 3x_1 + x_2 + x_3 &\leq 52, \\ 2x_1 + 4x_2 + 2x_3 &\leq 70, \\ 6x_1 + 5x_2 + 2x_3 &\geq 100, \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

The attached spreadsheet *Q6Spreadsheet1* is set up to solve this problem using the Solver tool.

(i) Explain why this problem cannot be solved by our graphical method. [2]

(ii) **Using the range names given,** state which range will form the "changing cells" and list the constraints that would need to be included in the Solver dialogue box, in order to solve this problem by setting cell B16 as the target to minimize. [4]

(iii) Describe briefly the nature of the *divisibility assumption* for a linear programming model, and give an example of a situation in which it may not be valid. [3]

(b) An oil refining company buys crude oil and distills it into gasoline, diesel and kerosene. Oil is available from the Middle East, Alaska and the North Sea. Because of the slightly different chemical make-up of the three oils, they cost different amounts per barrel and generate different amounts of the three fuels. The cost per barrel in pounds and the amounts of each fuel-type generated by each type of crude in kg, is shown in the attached spreadsheet *Q6Spreadsheet2*.

(Note that the spreadsheet gives the Number of Barrels in 000s and hence the Total Cost, BUdget, Amounts Achieved and the Goals are all in 000s also.)

The refinery has £4 million to spend on crude oil and has set the following goals for its production of fuel.

GOAL 1: At least 2.5million kg of gasoline is to be produced.

GOAL 2: At least 2million kg of diesel is to be produced.

GOAL 3: At least 2million kg of kerosene is to be produced.

After determining that all three goals cannot be met with their existing budget constraint, the refinery try to meet these goals one by one in priority order, Goal 1, Goal 2 and lastly Goal 3.

(i) **Using the range names given,** describe how you would fill in the Solver dialogue box to check that this problem has no feasible solution. You should indicate the ranges that would form the Target Cell and Changing Cells, list the constraints used and the Options that must be checked. (There is no need to explain where to find the Solver Tool in Excel's menu system.) [4]

(ii) **Using the range names given,** describe how to run the Goal Programming process to get as close as possible to each of the three goals in order of priority. Include in your description the target cell, changing cells and constraints used at each stage and mention any changes that must be made to the spreadsheet between stages.
NOTE: DO NOT attempt to solve the problem or produce any values. Simply describe the process. [12]

End of Question 6.