UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2003

COMPUTING

CIS102w Mathematics for Computing

Duration: 3 hours

Date and time:

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*There are **TEN** questions on this paper. Answer **ALL** of them.*
*Full marks will be awarded for complete answers to **ALL** **TEN** questions.*
Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1 (a) Working in base 2 perform the following calculation, showing all your working:

$$110111_2 + 10101_2 - 100001_2.$$

[3]

(b) Express the following hexadecimal number as a decimal number: $(A32.8)_{16}$. [3]

(c) Convert the following decimal number into base 2, showing all your working: $(213)_{10}$. [2]

(d) Express the recurring decimal 0.8181... as a rational number in its simplest form. [2]

Question 2 (a) List the elements of the following finite sets:

$$(i)\ \{2r-3 \ : \ r \in \mathbb{Z} \text{ and } 2 \le r \le 7\}$$
$$(ii)\ \{2^{r-1} \ : \ r \in \mathbb{Z} \text{ and } -2 < r < 4\}.$$

[4]

(b) Let A, B, C be general subsets of a universal set U

(i) Draw and shade on a Venn diagram the region represented by the set

$$(A \cup B)' \cap C.$$

[2]

(ii) Given $U = \{p, q, r, s, t, u, v, w\}$, $A = \{p, q, s\}$, $B = \{r, s, t, v\}$, $C = \{p, s, u, v\}$ find the elements of the set given by $(A \cup B)' \cap C$. [2]

(iii) Describe in set notation the region corresponding to $\{r, t, q\}$. [2]

Question 3 (a) Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions:

$$p : n \text{ is even}, \quad q : \ n > 15.$$

Draw up truth tables for the following statements and find the values of n for which they are true:

(i) (i) $p \vee \neg q$ (ii) $\neg p \wedge q$

(ii) Use truth tables to find a statement that is logically equivalent to $\neg p \rightarrow q$. [6]

(b) Let p, q be the following propositions:

$$p: \text{ this apple is red}, \quad q: \text{ this apple is ripe.}$$

Express the following statements in words as simply as you can:
(i) $p \to q$ (ii) $p \wedge \neg q$.
Express the following statements symbolically:
(iii) This apple is neither red nor ripe.
(iv) If this apple is not red it is not ripe. [4]

Question 4 (a) The function $f : \mathbb{R} \to \mathbb{Z}$ is defined by the rule $f(x) = \lfloor \frac{x}{3} \rfloor$.

(i) Find $f(2)$, $f(10)$.

(ii) Find the range of f.

(iii) Find the set of pre-images of 1.

(iv) Say, with reason, whether or not f is invertible. [6]

(b) Copy and complete the following table of values for the functions $g(x) = \log_3 x$ and $h(x) = \sqrt[3]{x}$.

x	1				81	
$g(x)$		1	2			5
$h(x)$ to 2 d.p.				3.00		

Is $\log_3 x = O(\sqrt[3]{x})$? Give a reason for your answer. [4]

Question 5 (a) Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and adjacency lists as follows:

$$\begin{array}{lcl} v_1 & : & v_2\ v_3\ v_4 \\ v_2 & : & v_1\ v_3\ v_4\ v_5 \\ v_3 & : & v_1\ v_2\ v_4 \\ v_4 & : & v_1\ v_2\ v_3. \\ v_5 & : & v_2 \end{array}$$

(i) List the degree sequence of G.

(ii) Draw the graph of G.

(iii) Find two distinct paths of length 3, starting at v_3 and ending at v_4.

(iv) Find a 4 cycle in G. [6]

(b) In the following cases either construct a graph with the specified properties or say why it is not possible to do so.

(i) A graph with degree sequence 3,2,2,1.

(ii) A simple graph with degree sequence 4,3,2,2. [4]

Question 6 (a) Let S be the set $\{2, 3, 4, 5, 6, 7\}$ and a relation $\mathcal{R}$ is defined between the elements of S by

$$\text{“}x \text{ is related to } y \text{ if } x - y \in \{0, 2, 4\}\text{”}.$$

(i) Draw the relationship digraph.

(ii) Determine whether or not $\mathcal{R}$ is reflexive, symmetric or transitive. In cases where one of these properties does not hold give an example to show that it does not hold.

(iii) State, with reason, whether $\mathcal{R}$ is a partial order or not. [8]

(b) Let X be the set of all 3 bit binary strings and Y be the set $\{0, 1, 2, 3\}$. The relationship $\mathcal{R}$ is defined to be the subset of X×Y where the elements x and y are related, $(x\mathcal{R}y)$, if y is equal to the number of zeros in x. List the elements of $\mathcal{R}$. [2]

Question 7 (a) A sequence is given by the recurrence relation

$$u_{n+1} = u_n + n \quad \text{and } u_1 = 0.$$

(i) Calculate u_3, u_4, and u_5. [2]

(ii) Use induction to prove that

$$u_n = \frac{n(n-1)}{2} \quad \text{for all } n \geq 1.$$

[5]

(b) Write the following in $\sum$ notation

$$1 + 4 + 7 + 10 + ... + (3n - 2).$$

Evaluate this when $n = 100$. [3]

Question 8 (a) In a tennis match two players, A and B, play up to 3 sets and the winner of the match is the first player to win a total of 2 sets. Each set is either won or lost, it cannot be drawn.

(i) Use a binary tree to model the possible outcomes of the match. [2]

(ii) The probability of A winning any given set is $\frac{3}{5}$ and the probability of B winning any given set is $\frac{2}{5}$. Find the probablities that: the match is won by A in 3 sets; the match lasts for 2 sets. Show your calculations clearly. [4]

(b) Construct a balanced binary search tree for an ordered list of 15 records, labelling them $1, 2, 3, ...15$ in your tree. What is the maximum number of comparisons a computer would have to make to match any existing record? [4]

Question 9 A pin number is formed by choosing an ordered sequence of 4 digits from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ where repetitions are not allowed but numbers beginning with 0 are.

(a) How many different numbers can be formed? [1]

(b) How many numbers will be (i) odd (ii) greater than or equal to 8000 (iii) odd and greater than or equal to 8000? [4]

(c) If A is the event that the number is odd and B is the event that the number is greater than or equal to 8000 calculate the following probabilities:

(i) $P(A)$

(ii) $P(B)$

(iii) $P(A \cap B)$. [3]

(d) Draw a Venn diagram to show the relationship between the sets A and B above.

(i) Shade the region containing the numbers which are even and ≥ 8000.

(ii) Put the number 7052 in the correct region on your diagram. [2]

Question 10 The following matrices are given as

$$A = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}.$$

(a) Show that AB and BA are not equal. [3]

(b) Find matrices C, D and E such that

(i) $B + C = A$

(ii) $BD = B$

(iii) $B - E = 2A$. [3]

(c) Write down the augmented matrix for the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 5 \\ x_1 + 0x_2 + x_3 &= 3 \\ 2x_1 + x_2 + 0x_3 &= 3. \end{aligned}$$

Solve this system of equations using Guassian Elimination. [4]

END OF EXAMINATION