UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2002

MATHEMATICS

MT52005A(M207) Classical Mechanics

Duration: 2 hours 15 minutes

Date and time:

Do not attempt more than FOUR questions on this paper.
Full marks will be awarded for complete answers to FOUR questions.
Electronic calculators may be used. The make and model should be specified on the script and the calculator must not be programmed prior to the examination.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1 A particle of mass m is suspended under gravity from a fixed point O by a light elastic string of natural length a. The string is extended by a distance ℓ below its natural length when the particle is not in motion. The particle is then displaced downward from this position and released. Given that the air resistance is $2mk$ times the speed of the particle, where k is a constant,

(a) show that the vertical displacement x at any time t measured from the equilibrium position, satisfies the equation

$$\ddot{x} + 2k\dot{x} + n^2 x = 0,$$

where

$$n = \sqrt{\frac{g}{\ell}}\ .$$

[9]

(b) By considering the solutions of this equation show that the motion of the system will only take the form of light damping if $k^2 < g/\ell$.

[5]

(c) Given that $k^2 = \alpha^2 n^2$, where $\alpha < 1$ and that the particle is released from rest at a distance $2a + \ell$ below the point O, show that

$$x = \ell e^{-\alpha n t}\left(\cos\left(1-\alpha^2\right)^{\frac{1}{2}} nt + \frac{\alpha}{(1-\alpha^2)^{\frac{1}{2}}}\sin\left(1-\alpha^2\right)^{\frac{1}{2}} nt\right).$$

[11]

Question 2 A particle of mass m is projected from ground level with initial horizontal velocity g/k and vertical velocity g/k where g is the gravitational acceleration and k is a constant. The motion of the particle is resisted by a medium which presents a resistance that is purely vertical and equal to mkv, where v is the vertical component of the velocity of the particle at time t.

(a) Show that the time taken to reach the greatest height is

$$\frac{1}{k}\log_e 2.$$

[9]

(b) Show also that the greatest height reached is

$$\frac{g}{k^2}\left(1 - \log_e 2\right).$$

[9]

(c) Find an expression for the energy lost by the particle during its ascent to the greatest height.

[7]

Question 3 A raindrop with an initial mass m_0 falls from rest under gravity through a cloud containing water vapour at rest. As it falls it increases its mass at a constant rate.

(a) Given that at time T_0 the mass of the drop is $5m_0$, show that the mass of the raindrop at time t is given by

$$m = m_0 \left(1 + \frac{4t}{T_0}\right).$$

[6]

(b) Show that the speed of the raindrop at time t is

$$v = \frac{g}{8}\left((T_0 + 4t) - \frac{T_0^2}{T_0 + 4t}\right).$$

[10]

(c) Hence find its speed after time T_0 and show that the distance travelled is

$$\frac{1}{8}gT_0^2\left(3 - \frac{1}{4}\log_e 5\right).$$

[9]

Question 4 A comet moves in a plane with its position described by polar coordinates (r, θ) relative to an origin fixed at the centre of the sun.

(a) Show that its radial component of acceleration is given by

$$\ddot{r} - r\dot{\theta}^2$$

and that its transverse component is

$$2\dot{r}\dot{\theta} + r\ddot{\theta}.$$

[7]