UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2002

COMPUTING AND INFORMATION SYSTEMS

IS53015A (CIS330) Mathematical Modelling in Management Science

Duration: 3 hours

Date and time:

Answer FIVE questions.

Full marks will be awarded for complete answers to FIVE questions.

There are 125 marks available on this paper.

Electronic calculators may be used. The make and model should be specified on the script. The calculator must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

Begin each question on a new page and number the question and parts.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

A company is interested in the relationship between sales productivity and the number of years a sales person has worked in the area.

Data were collected and recorded in the attached Excel spreadsheet. The option Add Trendline (polynomial of order 2) was used to find the implied relationship. The Excel output is shown on the spreadsheet.

(a) According to these results, what is the predicted sales productivity of a salesperson with 12.5 years experience? [3]

(b) What could you say about the sales productivity of a salesperson with 30 years experience? [2]

(c) Describe and interpret this model. [5]

(d) Suppose you were interested in defining the range of years of experience, in which there is an increase in productivity.

(i) How would you proceed analytically? [5]

(ii) How would you use Excel to locate this range? [5]

(e) What are the limitations of this approach? Could a company use it as a guideline for assessing the productivity of its salesforce? [5]

Question 2

Northdown Electronics makes components for a major manufacturer of aircraft engines. The manufacturer notifies the Northdown sales office each quarter of its requirements for the next three months. These can vary quite widely according to the type of engine that the manufacturer is producing. The table below shows the demand for two components, coded X and Y, for the coming three months.

Demand	Jul	Aug	Sep
X	1000	3000	5000
Y	1000	500	3000

Northdown has a total production capacity for X and Y taken together of 6000 units per month and sufficient storage capacity to hold a total of at most 2500 units after the demand for that month has been met (here we assume for simplicity that production takes place during the month and demand is met at the end of the month). The forecasted unit production costs in each month are given in the table below.

Production cost per unit	Jul	Aug	Sep
X	£20.00	£21.50	£19.50
Y	£10.20	£10.80	£10.00

The average holding cost per unit for each component at the end of any month is estimated at 2% of the production costs for that component during the month (this cost includes the cost of storage and money tied up in inventory). At the start of July, there will be no unit of either component in inventory. Northdown requires a production schedule which minimises the total production and holding costs for components X and Y for the months of July to September.

This problem has been modelled on the attached spreadsheet as a linear programming problem. The solution shown is optimal and has been found using the Solver tool in Excel.

(a) Describe the model used. You may use (if you wish) the range names shown in the textbox on the spreadsheet and refer to appropriate Excel functions. State carefully the objective and all constraints, including any constraints not shown on the spreadsheet. [16]

(b) Consider how the demand for each component is satisfied in each of the months July, August, September. How much of each monthly demand is met from inventory and how much is met from current production? [5]

(c) Suppose that Northdown is able to raise the maximum total production capacity in September (only) to 6500 units, while maintaining the production costs given in the table above. Can the company use this extra production capacity to reduce the total cost of meeting the given demand? Justify your answer. [4]

Question 3

(a) A local health authority must determine the allocation of its ambulance fleet for the following year. It costs £5000 per year to run an ambulance. The area covered by the authority is divided into two districts. Let x_1, x_2 be the number of ambulances assigned to district 1 and district 2 respectively (where fractional numbers represent an ambulance that is shared between the two districts and may operate in either). The average response time (in minutes) for an ambulance is $40 - 3x_1$ in district 1 and $50 - 4x_2$ in district 2. The local authority has three goals, which we list according to their priority:

- Goal 1: At most £100,000 per year should be spent on the ambulance service;
- Goal 2: Average response time in district 1 should be at most five minutes;
- Goal 3: Average response time in district 2 should be at most five minutes.

A solution to the problem is presented in the attached spreadsheet.

(i) Describe the model used. [7]

(ii) Describe the solution. [6]

(b) Your department needs to decide which of two secretarial candidates to hire. You have identified that three objectives are important to the decision: personality, typing ability and intelligence. You have also assessed the pairwise comparison matrix for these three objectives and the score of each candidate on each objective. Suppose you followed the Analytic Hierarchy Process and modelled the problem as shown in the spreadsheet attached.

(i) Describe the approach which was taken and its basic assumptions. [7]

(ii) Describe the results and their implication. [3]

(iii) What is the main limitation of this analysis? [2]

Question 4

A construction project consists of twelve different activities, denoted by the letters A, B, ...,L. The table below gives the duration of each activity (in days) and the activities (if any) which must precede it. The project will be completed when activity L is complete.

Activity	Duration (days)	Preceding Activities
A	3	-
B	2	-
C	2	A
D	6	A
E	3	B
F	5	C
G	2	E
H	3	E
I	6	D,G
J	7	D,G,H
K	3	F,I
L	2	J,K

(a) Construct a Critical Path Analysis-network to model this project [8]

(b) Give the events in this project an acylic labelling. Calculate the early event time ET(v) and the late event time LT(v) for each event v and record these times on your diagram. Tabulate the total float time for each activity. [8]

(c) What is meant by saying that an activity is **critical**? Say briefly why it is important for management to identify all the critical activities in a project. Determine the critical path(s) in this network and give the shortest completion time for the project. [4]

(d) Suppose each critical activity can be speeded up by at most one day. Show that if no non-critical activity is also speeded up, the shortest completion time for the project can be reduced by at most four days. Identify four critical activities such that if each is speeded up by one day, then this reduction is achieved. [5]

Question 5

An oil company produces three grades of petrol, called super, regular and economy, by mixing two crude oils, C_1 and C_2. The amount of each crude oil required to make one barrel of petrol of each grade is given in the following table.

	Super	*Regular*	*Economy*
Crude oil C_1	0.7	0.5	0.4
Crude oil C_2	0.3	0.5	0.6

The company is opening a new refinery with a maximum production capacity of 10,000 barrels per week. There are at most 6000 barrels of C_1 and at most 5000 barrels of C_2 available to the refinery each week. The company estimates it can make a profit per barrel of £35 on the sale of super grade, £25 on regular grade and £20 on economy grade petrol. It wants to know the number of barrels of each type of petrol it should produce per week at this refinery to maximise its profit.

(a) Explain briefly how this problem can be modelled by the following linear programming problem. You should say what the decision variables represent and how the objective function and constraints are derived. [5]

Find $x_1, x_2, x_3 \in \mathbf{R}$ to maximize $z = 35x_1 + 25x_2 + 20x_3$ subject to:

$$0.7x_1 + 0.5x_2 + 0.4x_3 \leq 6000 \quad (0.1)$$
$$0.3x_1 + 0.5x_2 + 0.6x_3 \leq 5000 \quad (0.2)$$
$$x_1 + x_2 + x_3 \leq 10000 \quad (0.3)$$
$$x_1, x_2, x_3 \geq 0. \quad (0.4)$$

(b) Describe briefly the main assumptions that must be made for this model to be valid. [5]

(c) Convert the constraints to a suitable form for the solution of this linear programming problem by the simplex algorithm. Give the initial basic feasible solution. (*Note: you are NOT required to perform any iterations of the simplex algorithm in this question.*) [3]

(d) After several iterations of the simplex algorithm, the following tableau is obtained, where x_5, x_6, x_7 are the slack variables in the constraints (0.1), (0.2) and (0.3) respectively.

Eq #	z	x_1	x_2	x_3	x_4	x_5	x_6	RS
0	1	0	0	2.5	25	0	12.5	280000
1	0	1	0	−0.5	5	0	−2.5	5000
2	0	0	0	0	1	1	−1	1000
3	0	0	1	1.5	−5	0	3.5	5000

(i) Explain why the solution represented by this tableau is optimal. [2]

(ii) Give the values of the decision variables that give an optimal solution for this problem and give the maximum value of z. Interpret this as a weekly production schedule for this refinery and give the maximum weekly profit they can expect to make on the sale of petrol. [4]

(iii) Give the values of the slack variables at the optimal solution. What extra information about the optimal solution do these values give? [4]

(iv) Suppose the refinery has an obligation to satisfy a weekly order for 1000 barrels of economy grade petrol and that it must meet this demand within its current weekly level of resources and production capacity. Would this cause their weekly profit to increase, decrease or stay the same? Give a reason for your answer. [2]

Question 6

Consider the following linear programming problem P.

Find $x_1, x_2 \in \mathbf{R}$ to minimize $z = x_1 + 2x_2$ subject to

$$\begin{aligned} x_1 - 2x_2 &\leq 2 \\ 2x_1 + x_2 &\geq 10 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(a) Express P as a maximization problem and prepare the constraints for solution by the simplex algorithm, introducing slack, surplus and artificial variables, as appropriate. [3]

(b) Give the augmented initial basic feasible solution to the revised problem. [2]

(c) Modify the objective function for the solution of P by the Big M method. Express the revised objective function in terms of the variables that are non-basic in the solution you gave in part (b) above. [3]

(d) State a rule for determining (i) the entering variable (EV) and (ii) the corresponding leaving variable (LV) in the iterative step of the simplex algorithm. Show how your rules are used to determine the first EV and the corresponding LV in this problem. [5]

(e) Complete the solution of P by the Big M method. State the optimal solution to P and the maximum value of z. [12]

Question 7

A company is considering marketing a new product. There are initially three possibilities: (A) to market it immediately, (B) to abandon the project, (C) to employ a market research organisation to carry out a survey on the likely demand for this product.

The company is undecided about which choice to make because it does not know whether demand will be heavy (H), moderate (M) or light (L). If demand is heavy the return will be 100, if moderate 50 and if light 10. (All units are in thousands of pounds). Marketing costs 30. The chances of H and M are assessed as 0.2 and 0.4 respectively.

The market research organisation can produce a favourable (f), neutral (n), or unfavourable (u), report. The company makes the following assessments:

$$\begin{array}{lll} p(f|H) = 0.9 & p(f|M) = 0.2 & p(f|L) = 0.1 \\ p(u|H) = 0 & p(u|M) = 0.1 & p(u|L) = 0.6 \end{array}$$

After the report of the survey has been received the company can choose between (A) and (B) above.

The cost of the survey is S.

(a) Draw an appropriate decision tree for this problem. [19]

(b) What is the maximum value of S that the company would be prepared to pay for the survey? [3]

(c) If $S = 3$ and the company employs the market research organisation what is the expected return overall if the report is favourable? [3]