UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

B. Sc. Examination 2002

COMPUTING AND INFORMATION SYSTEMS

IS53014A (CIS331) Introduction to Mathematical Modelling in Management Science

Duration: 2 hours 15 minutes

Date and time:

Answer FOUR questions.

Full marks will be awarded for complete answers to FOUR questions.

There are 100 marks available on this paper.

Electronic calculators may be used. The make and model should be specified on the script. The calculator must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

GRAPH PAPER is required for this examination.

Begin each question on a new page and number the question and parts.

THIS EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Question 1

A company is interested in the relationship between sales productivity and the number of years a sales person has worked in the area.

Data were collected and recorded in the attached Excel spreadsheet. The option Add Trendline (polynomial of order 2) was used to find the implied relationship. The Excel output is shown on the spreadsheet.

(a) According to these results, what is the predicted sales productivity of a salesperson with 12.5 years experience? [3]

(b) What could you say about the sales productivity of a salesperson with 30 years experience? [2]

(c) Describe and interpret this model. [5]

(d) Suppose you were interested in defining the range of years of experience, in which there is an increase in productivity.

(i) How would you proceed analytically? [5]

(ii) How would you use Excel to locate this range? [5]

(e) What are the limitations of this approach? Could a company use it as a guideline for assessing the productivity of its salesforce? [5]

Question 2

Northdown Electronics makes components for a major manufacturer of aircraft engines. The manufacturer notifies the Northdown sales office each quarter of its requirements for the next three months. These can vary quite widely according to the type of engine that the manufacturer is producing. The table below shows the demand for two components, coded X and Y, for the coming three months.

Demand	Jul	Aug	Sep
X	1000	3000	5000
Y	1000	500	3000

Northdown has a total production capacity for X and Y taken together of 6000 units per month and sufficient storage capacity to hold a total of at most 2500 units after the demand for that month has been met (here we assume for simplicity that production takes place during the month and demand is met at the end of the month). The forecasted unit production costs in each month are given in the table below.

Production cost per unit	Jul	Aug	Sep
X	£20.00	£21.50	£19.50
Y	£10.20	£10.80	£10.00

The average holding cost per unit for each component at the end of any month is estimated at 2% of the production costs for that component during the month (this cost includes the cost of storage and money tied up in inventory). At the start of July, there will be no unit of either component in inventory. Northdown requires a production schedule which minimises the total production and holding costs for components X and Y for the months of July to September.

This problem has been modelled on the attached spreadsheet as a linear programming problem. The solution shown is optimal and has been found using the Solver tool in Excel.

(a) Describe the model used. You may use (if you wish) the range names shown in the textbox on the spreadsheet and refer to appropriate Excel functions. State carefully the objective and all constraints, including any constraints not shown on the spreadsheet. [16]

(b) Consider how the demand for each component is satisfied in each of the months July, August, September. How much of each monthly demand is met from inventory and how much is met from current production? [5]

(c) Suppose that Northdown is able to raise the maximum total production capacity in September (only) to 6500 units, while maintaining the production costs given in the table above. Can the company use this extra production capacity to reduce the total cost of meeting the given demand? Justify your answer. [4]

Question 3

Bangers is a small company which buys second hand cars and trucks to repair and refurbish for resale. Each vehicle must be processed in the engine shop, the body shop and the paint shop. The average number of hours work which each type of vehicle requires in each shop is shown in the table below.

	work-hours per car	*work-hours per truck*
Engine shop	24	15
Body shop	40	40
Paint shop	20	30

Bangers have at most 1320 work-hours available per week in the engine shop, at most 2400 work-hours per week in the body shop and at most 1500 work-hours per week in the paint shop. Each car contributes on average £1500 to profit and each truck contributes on average £2000 to profit. We assume there is an unlimited supply of cars and trucks available to the firm for refurbishment and that it can resell all the vehicles it processes. Bangers requires a weekly production schedule that will maximise its profit.

(a) Formulate the problem of determining the number of cars and the number of trucks that the firm should process each week as a linear programming (l.p.) problem. Define the decision variables and say briefly how the objective function and each constraint is obtained. [7]

(b) Plot the constraint lines for this l.p. problem, either using the graph paper provided or by drawing a neat sketch graph. Indicate the feasible region clearly on your graph. [6]

(c) By plotting a suitable objective line, or otherwise, determine the optimal solution. [4]

(d) Interpret the optimal solution to this problem as a weekly production schedule, giving the maximum profit the firm could expect to make with this schedule. Say why it is not possible for them to make a higher profit with the given resources. [4]

(e) Comment briefly on the validity of of the assumptions of proportionality, additivity and divisibility with regard to this particular problem. [4]

Question 4

The vertices $v_1, v_2, \ldots, v_n$ in a network N represent n sites. Each pair of sites can be joined by a direct link at a positive cost proportional to a weight $w(v_i, v_j)$ associated with the edge $v_i v_j$ in N. It is required to construct a subnetwork T of N such that every pair of vertices of N are connected by a path in T and the total weight on the edges of T is as small as possible.

(a) Why is the required subnetwork T a *spanning tree* of N? [3]

(b) Describe the steps in *Prim's Algorithm* for finding a minimum weight spanning tree in a network. [4]

(c) The table below gives weights on the edges of a network N with vertices $v_1, v_2, \ldots, v_8$.

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
v_1	-	3	5	4	2	4	1	6
v_2	3	-	3	6	5	1	4	4
v_3	5	3	-	3	4	5	2	1
v_4	4	6	3	-	6	7	2	5
v_5	2	5	4	6	-	5	8	3
v_6	4	1	5	7	5	-	6	7
v_7	1	4	2	2	8	6	-	5
v_8	6	4	1	5	3	7	5	-

(i) Describe how you would implement Prim's algorithm on this table of weights to find a minimum weight spanning tree in N, starting from the vertex v_1. Explain how you would select the next *two* vertices and edges to add to the tree and how you would modify the table of weights at each step. Complete the solution by Prim's algorithm (without further explanation) and draw the tree you obtain. State its total weight. [12]

(ii) Explain why there is just one alternative optimal solution, and draw the corresponding minimal spanning tree. [6]

Question 5

(a) A local health authority must determine the allocation of its ambulance fleet for the following year. It costs £5000 per year to run an ambulance. The area covered by the authority is divided into two districts. Let x_1, x_2 be the number of ambulances assigned to district 1 and district 2 respectively (where fractional numbers represent an ambulance that is shared between the two districts and may operate in either). The average response time (in minutes) for an ambulance is $40 - 3x_1$ in district 1 and $50 - 4x_2$ in district 2. The local authority has three goals, which we list according to their priority:

- Goal 1: At most £100,000 per year should be spent on the ambulance service;
- Goal 2: Average response time in district 1 should be at most five minutes;
- Goal 3: Average response time in district 2 should be at most five minutes.

A solution to the problem is presented in the attached spreadsheet.

(i) Describe the model used. [7]

(ii) Describe the solution. [6]

(b) Your department needs to decide which of two secretarial candidates to hire. You have identified that three objectives are important to the decision: personality, typing ability and intelligence. You have also assessed the pairwise comparison matrix for these three objectives and the score of each candidate on each objective. Suppose you followed the Analytic Hierarchy Process and modelled the problem as shown in the spreadsheet attached.

(i) Describe briefly the steps in the calculation to decide which candidate to hire. [7]

(ii) Describe the results and their implication. [3]

(iii) What is the basic assumption behind this analysis? [2]

Question 6

A construction project consists of twelve different activities, denoted by the letters A, B, ...,L. The table below gives the duration of each activity (in days) and the activities (if any) which must precede it. The project will be completed when activity L is complete.

Activity	Duration (days)	Preceding Activities
A	3	-
B	2	-
C	2	A
D	6	A
E	3	B
F	5	C
G	2	E
H	3	E
I	6	D,G
J	7	D,G,H
K	3	F,I
L	2	J,K

(a) Construct a Critical Path Analysis-network to model this project [8]

(b) Give the events in this project an acylic labelling. Calculate the early event time ET(v) and the late event time LT(v) for each event v and record these times on your diagram. Tabulate the total float time for each activity. [8]

(c) What is meant by saying that an activity is **critical**? Say briefly why it is important for management to identify all the critical activities in a project. Determine the critical path(s) in this network and give the shortest completion time for the project. [4]

(d) Suppose each critical activity can be speeded up by at most one day. Show that if no non-critical activity is also speeded up, the shortest completion time for the project can be reduced by at most four days. Identify four critical activities such that if each is speeded up by one day, then this reduction is achieved. [5]