Quadrilateral Meshes for the Registration of Human Brain Images

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1 Introduction

Finite element analysis (FEA) is a powerful tool for numerically solving differential equations or variational problems that arise during structural modeling in engineering and the applied sciences. A key feature of FEA is the specification of a *mesh* over the problem domain. Triangular meshes have been extensively investigated by the meshing community, and their theoretical properties are now well understood, [1]; however, the generation of good quadrilateral meshes is not as well understood. Most automated mesh generators cater to the construction of meshes for structural engineering applications, [2], but these are typically optimized for much simpler shapes than those encountered in biology.

This paper is mainly concerned with the generation of quadrilateral meshes from magnetic resonance (MR) images of the human brain, and the effect of mesh parameters on the accuracy and performance of a particular implementation of the FE-based non-rigid registration method described in [3] and [4]. We present the main features of our new algorithm for generating strictly convex quadrangulations from polygonal domains, describe methods for generating such meshes from image data, and discuss the performance of these meshes when applied to an image registration problem.

2 Convex Quadrangulation

A quadrangulation $\mathcal{Q}$ of a polygonal region $\mathcal{R}$ is a decomposition of $\mathcal{R}$ into a set of non-overlapping quadrilaterals. Many algorithms for quadrangulating polygons adopt an indirect approach: the polygon is first triangulated, and then the triangulation is converted into a quadrangulation of the polygon. Those algorithms that guarantee a convex quadrangulation, i.e., a quadrangulation in which all four angles of every quadrilateral are less than or equal to 180^o, are suitable for FE-based applications.

By modifying one of the algorithms in [5], we have obtained a new algorithm for converting triangulations of general polygonal regions into *strictly* convex quadrangulations (i.e. ones in which all four angles of every quadrilateral are *less* than 180^o) using a bounded number of Steiner points and runtime linear in the number of triangles. Its underlying premise is to systematically group and quadrangulate a few triangles at a time, so that no isolated triangles will remain at the end of the process. Its bounds are significantly better than those of algorithms that also produce strictly convex quadrangulations of bounded size, [6].

3 Quadrilateral Meshes from Images

Our method for generating quadrilateral meshes from two-dimensional (2D) image data consists of three steps: contour generation, triangulation, and quadrangulation. During the first step, a segmented version of the image is obtained, and a closed polygonal curve is generated for every boundary defining a distinct anatomic feature of the segmented image (Figure 1a). For every polygonal region that results from the first step, we generate a triangular mesh in the second step such that the mesh of each polygonal region coincides with the mesh of neighboring regions at the boundary. As a result, every triangle belongs to only one polygonal region, and the set of all meshes is

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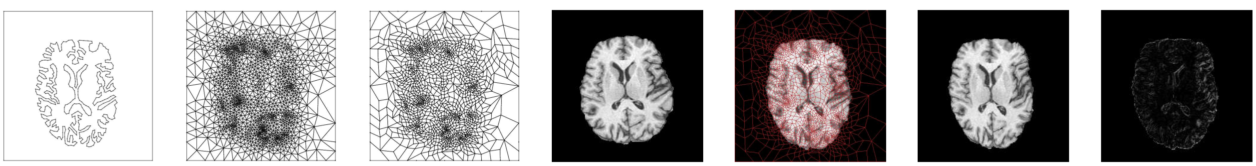
a mesh of the domain defined by all feature contours and their interiors (see Figure 1b). Finally, using our new algorithm at the third step, we convert the triangulation of each polygonal region into a quadrangulation, so that the resulting quadrangulations define a quadrilateral mesh of the entire domain, and every quadrilateral belongs to only one polygonal region (Figure 1c). Because our quadrangulation algorithm preserves local variations in element size, the resulting quadrilateral meshes reflect the anatomical features highlighted by the input triangulation.

4 Results and Conclusions

In order to evaluate the quadrilateral meshes generated by our algorithm, we performed a quantitative, registration-based analysis using a pair (A, B) of 2D images of the human brain. A was a coronal MR image with dimensions equal to 256x256 (Figure 1d) and B was the result of applying a cubic polynomial warp to A (Figure 1e). Next, we used the method described in Section 3 to generate quadrilateral meshes of A and B. We then registered A to B using a variety of triangular and quadrilateral meshes with the implementation of Gee and Bajcsy's FE-based elastic registration method in the Insight Segmentation and Registration Toolkit (ITK), [7].

Examination of the results (Figures 1e-g) demonstrates that the quadrangulated meshes have slightly more than half the number of elements of their triangular counterparts, but that by modifying the number of integration points used to sample the elements, we can achieve similar sampling frequencies throughout both mesh types and compare their performance. The error associated with the triangular meshes is comparable to that of the quadrilateral meshes, supporting the propagation of regional characteristics within the triangulation to the subsequent quadrangulation. The experiments also emphasize the fact that uniform grids require many more elements than adaptive meshes to produce results with similar accuracy, and that resolving this issue involves using denser grids that markedly increase runtime.



(a) (b) (c) (d) (e) (f) (g)

Figure 1: (a) Contours of a 2D segmented brain image. (b) A triangular mesh generated using the contours in (a). (c) A quadrilateral mesh constructed from the triangulation in (b). (d) A coronal MR image of a human brain. (e) The quadrilateral mesh in (c) overlaid on a synthetic image generated by warping the image in (d) using a cubic polynomial. (f) The image resulting from the FE-based registration of the brain depicted in (d) to its synthetically deformed version in (e), using the mesh shown in (e). (g) The registration error is depicted here using the difference image obtained by subtracting the registered image in (f) from the target image in (e).

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