Boundary Tracking for Both the Simple Cubic and the Face-Centered Cubic Grids

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Biomedical imaging is an example of an area greatly benefited from mathematics, computer science and engineering. In a typical three-dimensional (3D) medical imaging application a series of steps have to be taken: acquisition, post-processing, and display. Modern medical imaging devices generate sets of numbers, or images, representing a given region of the body. In general, these sets of numbers are discrete representations of real objects obtained by either direct sampling the objects or by using a reconstruction algorithm. The final set of numbers is arranged in a 3D grid, we refer to the set of points to which the samples of an object are located as a grid. Every point in the grid is associated to a volume element or voxel; moreover, a voxel can be referred to by the grid point with which it is associated. A common practice is to assign a real value to each voxel, i.e., gray value, such that the given value is proportional to some physical property of the body imaged. A common grid used in biomedical imaging is the simple cubic grid, whose associated voxels are cubic voxels, because is the easiest to implement in current computers.

The discrete approximation of the object has to be further processed after acquisition (e.g., filtered and segmented). One reason for post-processing the discrete object is that the image is corrupted by noise and artifacts introduced by either the physics of the imaging device, the acquisition techniques, the limitations of the device or the inaccuracies of reconstruction algorithms. Another reason for post-processing is the wish, or need, to observe only a section of the object imaged, e.g., the desire to observe an organ. In order to carry out the post-processing operations is also important to consider the arrangement of the voxels in space, the way voxels are adjacent to each other and the different way gray values are assigned to voxels. For example, in the simple cubic grid, voxels can be considered to be adjacent in several ways: face, face-or-edge or face-or-edge-or-vertex. The final result of the post-processing operations is a subset of voxels, or segmented image, that conjecturally approximate more precisely the object to be analyzed.

Finally, the resulting subset of voxels is typically further processed to make it appropriate for displaying on a computer screen. One of the typical techniques to create final displays is surface rendering. In this technique we assume that the subset of voxels is an accurate approximation of the volume of the object to be analyzed and that the interior of the object is of no interest. Consequently, the surface enclosing the volume is enough to represent the object on a computer screen. In the case of cubic grids, the surface is a collection of square faces that define the boundary between the interior of the object and its exterior (the final representation on the computer screen is created using standard computer graphics techniques). Several techniques have developed to obtain the collection of square faces that represent the surface of a subset of cubic voxels, also known as boundary tracking algorithms. Notably, to obtain such collection is also essential to consider the properties of the arrangement of the grid points, the relation between its associated voxels and the gray values assigned to the voxels.

The increasing use of digital computers to carry out the processing of information has led to the development of a mathematical theory to deal with the information represented in a discrete fashion, i.e., geometry of digital spaces. The geometry of digital spaces is a theory capable of dealing with various grids (or tesellations of space), adjacencies between grid points, different assignments of values to voxels and operations performed with them. An important result of the use of the theory of digital spaces is that there are other grids that are more convenient to discretize an object and to carry out image processing. One of such grids is the face-centered cubic grid, i.e., fcc grid. The voxels associated with the points in the fcc grid are rhombic dodecahedra (polyhedra with 12 identical rhombic faces). Significantly, the fcc grid is a subset of the simple cubic grid, a fact that allows to take advantage of some existing algorithms for the simple cubic grid. We present a boundary tracking algorithm capable of producing boundaries for 3D binary images modeled either with cubic voxels or rhombic dodecahedral voxels.

Furthermore, boundaries utilizing cubic voxels have some undesirable properties. One of them is that the 90-degree angle between the faces of a cubic voxel results in a ”blocky” appearance when displayed on a computer screen. We diminish this problem by using rhombic dodecahedral voxels.