Determining Bi-Plane Imaging Geometry for Reconstructing 3-D Vascular Structures

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1 Overview

Vascular disease (such as blockage) has been a major cause of death in the United States for a long time. Effective treatment and diagnosis procedures for this type of diseases heavily rely on accurate 3-D images of the interested vessel structures. Currently dominating approach for obtaining the rapidly moving vascular structures is to use one or more projections (generated by image intensifier-TV (II-TV)) to reconstruct the 3-D images (as the more advanced MRI and CT cannot yet provide the necessary time resolution). Thus a key problem in such reconstructions is to determine the geometry of the imaging systems (i.e., the rotation matrix R and translation vector t relating two single-plane imaging systems).

As a promising approach, bi-plane imaging has received considerable attention in recent years and a number of techniques have been developed for 3-D reconstruction [2, 4, 5, 6]. A common feature of these techniques for determining imaging geometry is to first identify a set of corresponding points in the two projections, and then convert the problem of geometry determination to certain non-linear optimization problem, and use either heuristic algorithms or general optimization packages to find a feasible solution to R and t. Unfortunately, these approaches in general can guarantee neither the quality of solutions nor the time efficiency, thus may not be suitable for online imaging systems.

In bi-plane imaging, two planar images are generated by projecting an X-ray beam (with a cone shape) from two different locations through the 3-D object to an image acquisition device (or screen). The beam source has a fixed distance D to the screen, and is the origin of the 3-D coordinate system associated with the image, where X and Y are the coordinates of the 2-D projection images, and Z is the normal of the screen. Due to a variety of reasons (such as movement of the beam source and data noise), the exact rotation matrix and translation vector between the two coordinate systems are often unknown. Rough estimation can be obtained by using technique in [6]. To accurately reconstruct the 3-D structures of tiny vessels, high precision imaging geometry is desired.

To provide a better solution, we reduce it to the following geometric problem: Given two sets of 2-D points $A = \{a_1, a_2, \cdots, a_n\}$ and $B = \{b_1, b_2, \cdots, b_n\}$ on two image screens with each pair of a_i and b_i being the approximation of the two projections of an (unknown) point p_i in 3-D space, also given the beam source (i.e., the origin) o_A of image A, find the most likely position of the beam source o_B and orientation of the coordinate system of B in the coordinate system of A. It is easy to see that in an ideal situation where a_i and b_i are the exact projections of p_i, it is sufficient to consider only a constant number of corresponding pairs. In practice, however, it is often difficult to find the exact positions for corresponding pairs (as most of the correspondance is done manually). Thus, a number of corresponding pairs are considered for increasing the accuracy.

In this paper, we present an efficient approach for solving the above problem. Our approach first reduces the imaging geometry determination problem to a problem of finding optimal cells in an arrangement of a set of surfaces in R^6. Based on interesting observations, we then simplify the rather complicated surfaces (which can not be analytically expressed) so that each of them can be implicitly expressed by an equation. The simplified surfaces are in general non-algebraic, indicating that directly computing the arrangement could be very challenging. To overcome this difficulty, we study the error sensitivity of each variable in the imaging geometry and use it to partition the feasible domain into smaller regions so that the topological structure of the arrangement in each region can be effectively captured by some lower dimensional (e.g., 2 or 3-D) arrangements. The curves and surfaces in these lower dimensional arrangements, although are still non-algebraic, have "nice" properties which enable us to efficiently find the optimal cell. Comparing with existing approaches, our technique achieves better accuracy (as suggested by our preliminary experimental results) and has bounded running time. Our technique can also be easily modified to remove a few outliers in point sets A and B to further improve the accuracy.

2 Main Ideas

Let $P = \{p_1, p_2 \cdots, p_n\}$ be the set of to-be-reconstructed points in 3-D space. Let p_i^a and p_i^b be the exact projections of p_i on the image screen of A and B, respectively. We define $\Delta = max_{i=1}^{n} max\{dist(a_i, p_i^a), dist(b_i, p_i^b)\}$, where $dist(.)$ is the Euclidean distance of two points.

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Note that p_i^a, p_i^b and Δ are all unknown.

To determine the best possible imaging geometry for B, we first guess a possible value, say δ, for Δ. Clearly if $\delta \geq \Delta$, then each p_i^a will be contained in the disk d_i centered at a_i and with radius δ. Thus p_i is contained in the round cone C_i apexed at the origin o_A and with d_i as the base. Given a solution G to the imaging geometry of B, we can project each cone C_i to the screen of B and form a sector S_i. If G is optimal, then each b_i will be contained in their corresponding sector S_i. Thus the optimality of G can be determined by counting the number $f_{in}(A, B, G)$ of points which are contained in their corresponding sectors. Therefore, finding the most likely imaging geometry can be reduced to finding a geometry G which maximizes the value $f_{in}(A, B, G)$.

To efficiently obtain the optimal G, we consider the relation of each point b_i and its corresponding sector S_i. Since S_i can be parameterized by the six variables of G, it seems possible to relate the location of b_i with G, and therefore functionally determine whether a point b_i is inside or outside S_i. When b_i moves inside S_i, it forces the corresponding G to move in the six dimensional solution space. The loci of G therefore form a region R_i in R^6, meaning that if G is inside R_i, b_i will be contained in S_i. Since each of the n b_i will generate a region, to maximize the value of $f_{in}(A, B, G)$ it is sufficient to determine the cell contained by the most number of R_i's.

To make this approach feasible, we need to determine the bounding surface of each R_i. Notice that each surface is the loci of G while moving b_i on the boundary of S_i. Thus we only need to consider the incidence of b_i with the two bounding rays of S_i, and use the condition for incidence to derive the function of the surface.

Unfortunately, the above approach does not work. This is because the surfaces cannot be analytically expressed. The computation of the surfaces requires to find the roots of polynomials with degree ≥ 6, which do not admit analytical formulas.

To overcome this difficulty, we approximate each round cone C_i by a convex facet cone FC_i with k facets. Depending on the location of G, the projection of FC_i will create up to k sectors, $S_i^1, S_i^2, \cdots, S_i^k$, on the screen of B, with each sector $S_i^j, 1 \leq j \leq k$, corresponding to a pair of edges on FC_i tangent to two planes crossing o_B. The facet cone FC_i also breaks each region R_i into $O(k)$ subregions $R_i^1, R_i^2, \cdots, R_i^k$, with each subregion R_i^j generated by a sector $S_i^j, 1 \leq j \leq k$. A nice property of these subregions is that their bounding surfaces can all be implicitly expressed.

Directly solving the maximization problem requires to compute the arrangements of those implicitly expressed surfaces in R^6, which seems to be quite difficult. To further simplify the problem, we study the influence of each variable to the solution. We show that when the 3-D object is roughly in the middle of the image systems (which is typically the case in practice), error is much less sensitive to the three translational variables than to the rotational variables.

Thus our approach is to first compute a rough estimation for G by using technique in [6], which gives us a small hyperbox H bounding the optimal solution of G. Based on the error-sensitivity of each variable, we then partition H into a set of regions. In each region, we select the leading rotational variable and one or two translational variables (e.g., t_y or t_y and t_x) in terms of error-sensitivity, and place a non-uniform grid (based on the error-sensitivity) in the subspace corresponding to those unselected variables. In each grid point, the six dimensional arrangement is reduced to a two or three dimensional arrangement. The following lemmas show some nice properties of such arrangemnts.

Lemma 1 *Let t_y, t_x and $\alpha \in \{\theta, \psi, \phi\}$ be three selected variables. Then, at any fixed grid point, the bounding surface S_i is monotone in the directions of t_x and t_y. Furthermore, the intersection of S_i and any plane parallel to the $t_x t_y$ plane is a straight line.*

Lemma 2 *Let $t_a, a \in \{x, y, z\}$ and $\alpha \in \{\theta, \psi, \phi\}$ be the two selected variables, then each curve is of the form $T_a = \frac{c_i \cos(\alpha) + d_i \sin(\alpha) + e_i}{g_i \cos(\alpha) + h_i \sin(\alpha) + j_i}, \alpha \in [0, 2\pi]$ or $[0, \pi]$, and can be break into up to 3 continuous pieces. Any pair of curves have no more than 4 intersections.*

The above lemmas enable us to find the optimal cell in each grid point by using some arrangement traversal algorithms [1, 3]. To find the "global" optimal, we determine an optimal point for each grid point and return the best as our solution.

To further reduce error, one can perform a binary search on δ to find the smallest value such that the corresponding arrangement contains at least one optimal solution. Since δ upper bounds the maximum error, our approach simultaneously achieves the accuracy and efficiency. Note that our algorithm can also be easily modified to remove a few outliers to further reduce error.

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