Methods for obtaining very thin tunnel-free discretizations of polyhedral surfaces

Reneta P. Barneva and Valentin E. Brimkov
Department of Mathematics and Computer Science, SUNY Fredonia

April 3, 2003

Having advanced algorithms for discretizing polyhedral surfaces (usually triangulations) is of significant practical importance, since for various applications it is sufficient to work with a polyhedral approximation of a given surface rather than with the surface itself. Often this is the only possibility since the surface may not be available in an explicit form. On the other hand, sometimes it is quite desirable that the obtained discretization be as thin as possible. Such a requirement may appear in medical imaging or terrain modeling, especially when very precise representation is needed under certain memory limitations. However, it has been very early recognized as a difficult task to obtain a tunnel-free discretization of a polyhedral surface, provided that every polygonal patch is approximated by a portion of the thinnest possible tunnel-free discrete plane. The difficulty is rooted in the discrete nature of the discrete plane primitives, which may cause appearance of tunnels at the discrete "edges" shared by two discrete polygons. The intersection of two discrete planes may indeed be too far from the intuitive idea of an edge and, in fact, may be a disconnected set of voxels. Moreover, under the above mentioned circumstances, it is usually hard to prove that a given algorithm produces tunnel-free discretization of a surface, even if it performs well in practice.

In this talk we will review three different methods for obtaining very thin discretizations of arbitrary polyhedral surfaces. The methods rest on different approaches. The first one is based on reducing the 3D problem to a 2D one by projecting the surface polygons on suitable coordinate planes, next discretizing the obtained 2D polygons, and then restoring the 3D discrete polygons [1]. The generated discrete polygons are portions of the thinnest possible discrete planes associated with the facets of the surface. The second method approximates directly every space polygon by a discrete one, which is again the thinnest possible, while the polygons' edges are approximated by the thinnest possible 3D straight lines [2]. The obtained discretization appears to be optimally thin, in a sense that removing an arbitrary voxels from the discrete surface leads to occurrence of a tunnel in it. The third method is based on introducing new fundamental classes of 3D lines and planes (called graceful) which are used to approximate the surface polygons and their edges, respectively [3].

All algorithms run in time that is linear in the number of the generated voxels, which are stored in a 2D array. Thus the algorithms are optimal regarding their time and space complexity. Another advantage of the proposed algorithms is that the generated 3D discrete polygons admit analytical description. The algorithms are simple and easy to implement.

The different algorithms have different advantages over each other, so that one may choose to apply a particular algorithm in accordance with the specific application. In the talk we will compare the algorithms' performance and discuss on related matters. Reminiscent work of other authors will be commented as well.

Finally, we will briefly consider an approach for discretizing higher dimensional surfaces [4]. We believe that this is interesting not only from theoretical perspective, but also regarding certain practical applications, such as 4D imaging related to PET scans and other dynamic medical images.

References

1. Barneva, R.P., V.E. Brimkov, Ph. Nehlig, Thin discrete triangular meshes, Theoretical Computer Science, Elsevier, 246 (1-2) (2000) 73-105.

2. Brimkov, V.E., R.P. Barneva, Ph. Nehlig, Minimally thin discrete triangulations, In: Volume Graphics, A. Kaufman, R. Yagel, M. Chen (Eds.), Springer Verlag (2000) Chapter 3, pp. 51-70.

3. Brimkov, V.E., R.P. Barneva, Graceful planes and lines, Theoretical Computer Science, Elsevier, 283 (2002) 151-170.

4. Brimkov, V.E., E. Andres, R.P. Barneva, Object discretizations in higher dimensions, Pattern Recognition Letters, Elsevier, 23 (6) (2002) 623-636.