Towards Surface Regularization via Medial Axis Transitions

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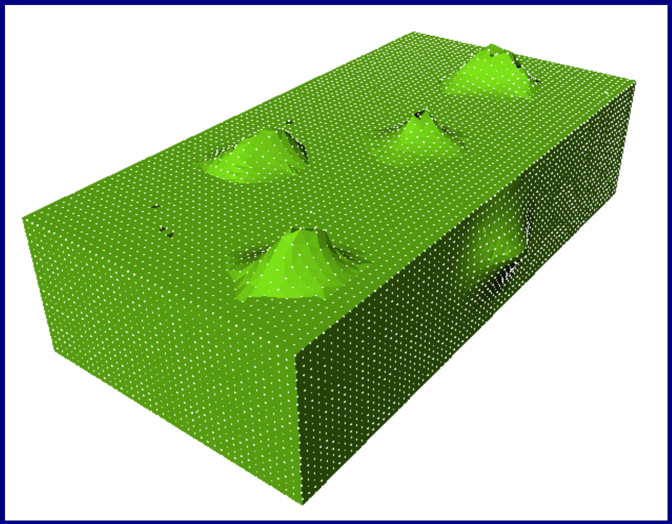
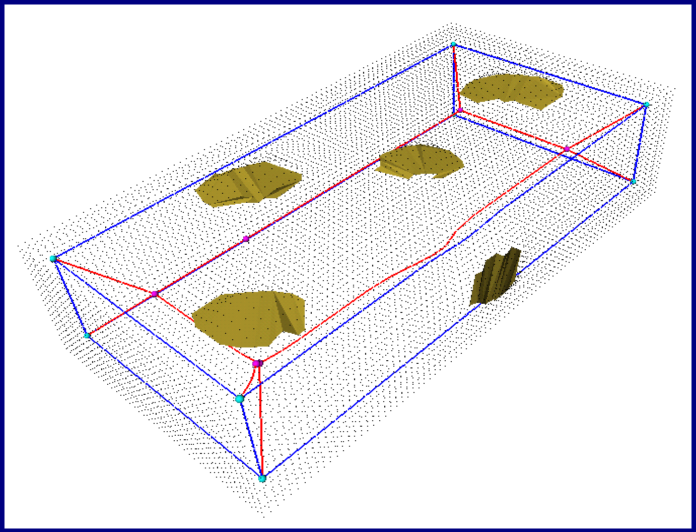
Outline

- Surface and object regularization – background
- Medial scaffold for 3D shape representation
- Transitions of the medial axis
- Algorithm
- Results
- Future work

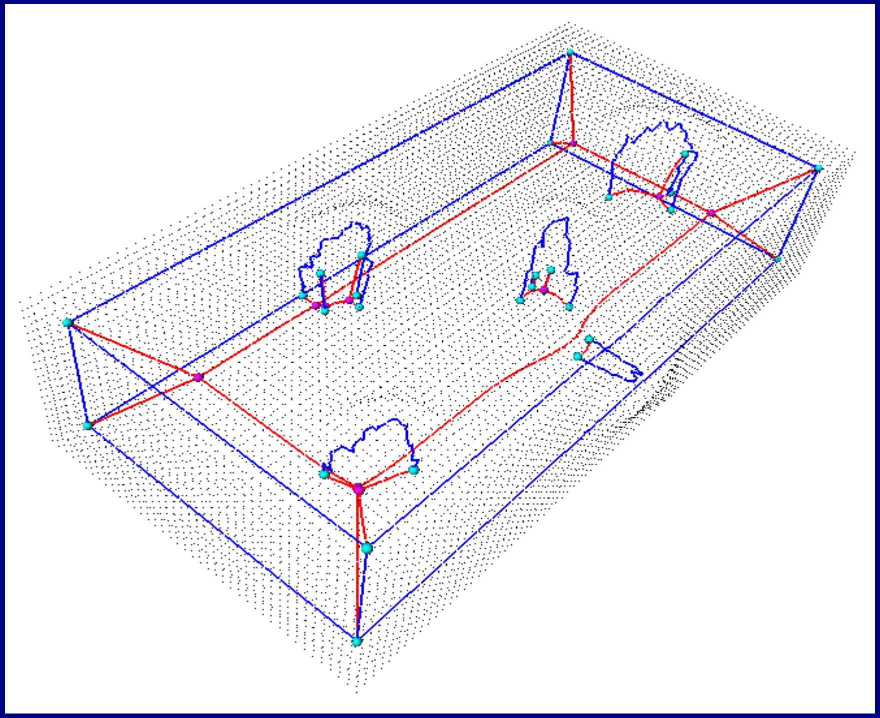
Surface and object regularization

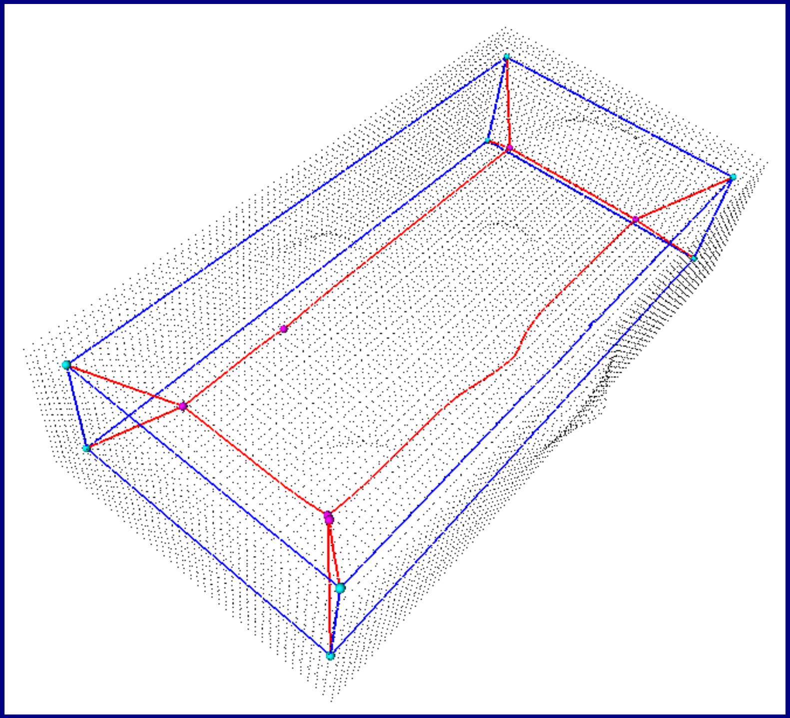




- Regularize shape: smooth away less important features while preserving significant ones in the vicinity.
- Classical smoothing cannot distinguish between features and noise.
- Recent anisotropic curvature smoothing methods are parameter sensitive, require a careful treatment of surface boundaries, and well-defined surface meshes (and normal fields) [Tasdizen02, Hildebrandt04].

Our goal: develop a hierarchical shape representation to elicit (geometric) regularization



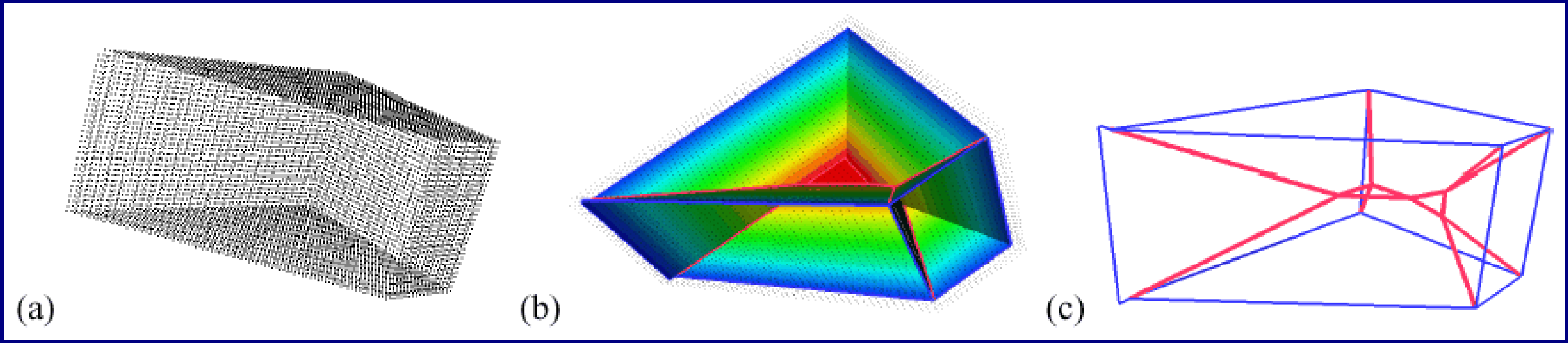


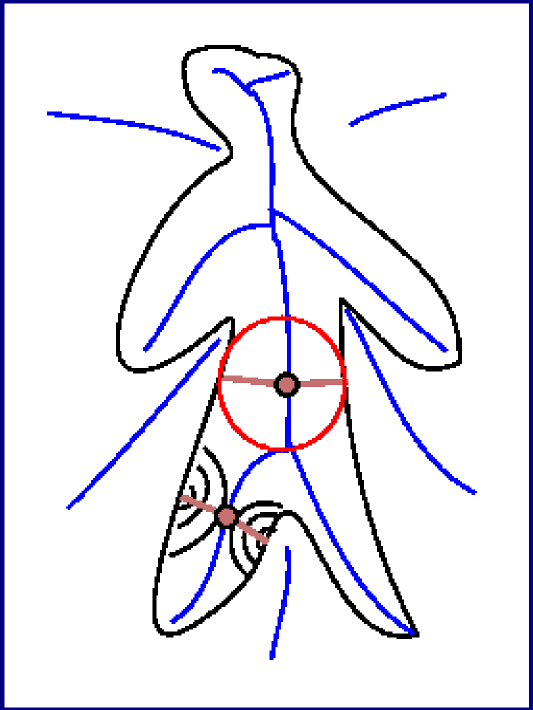
- Study the dynamics of the Medial Axis (MA) under perturbations.
- Requires a representation of the MA as a graph.

Achievements

- Take input as unorganized point clouds or polygonal meshes.
- Can deal with objects under real scans, process partial surfaces.
- Robust under different resolutions, acquisition conditions.
- Do not require a closed surface mesh or voxelization, *i.e.*, no inside/outside, skeleton can be a graph with loops (not a tree).
- Do not over-simplified the extracted skeletal graph.
- Regularization: based on a comprehensive analysis of the the medial structure w.r.t. topological changes under shape perturbations.

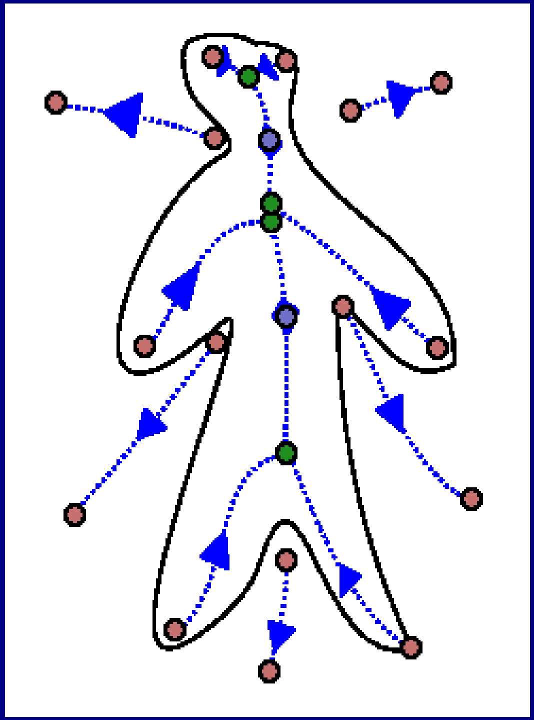
Shape representation: From the Medial Axis to the Medial Scaffold





Blum (1960's & 70's):
Propagation vs.
Contact with disks

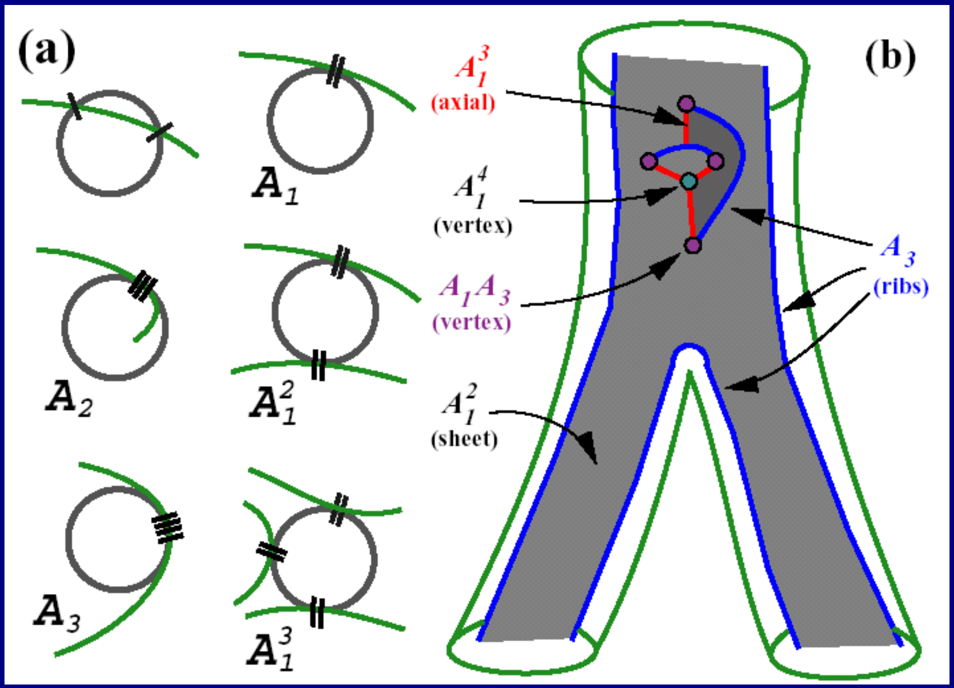
2000+:
Shock singularities
Contact typology
(Kimia, Giblin, *et al.*)



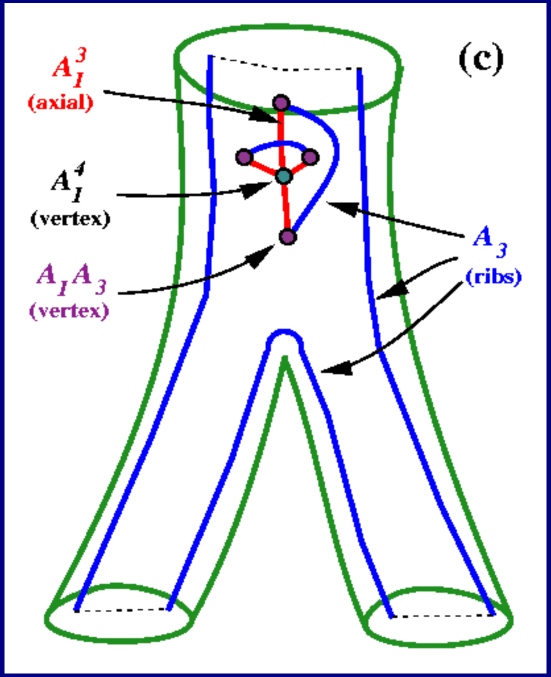
Shape representation: From the Medial Axis to the Medial Scaffold

- 3D: Five types of points from contact theory [Giblin-Kimia PAMI04]:
  - Sheet: A_1^2
  - Links: A_1^3 (**Axial**), A_3 (**Rib**)
  - Nodes: A_1^4 (Voronoi vertices) , A_1A_3

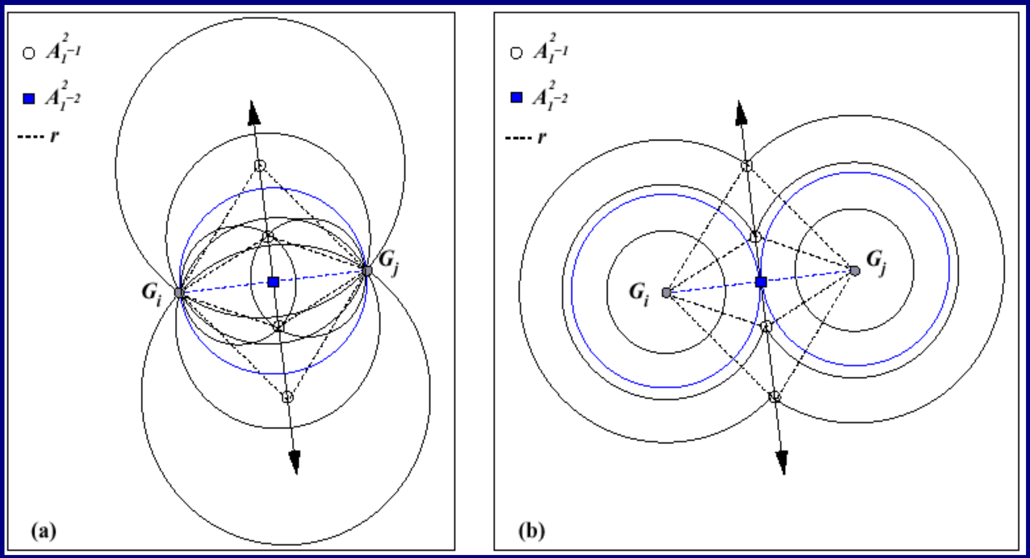
$\mathbf{A}_k^n$: contact at *n* distinct points, each with *k+1* degree of contact

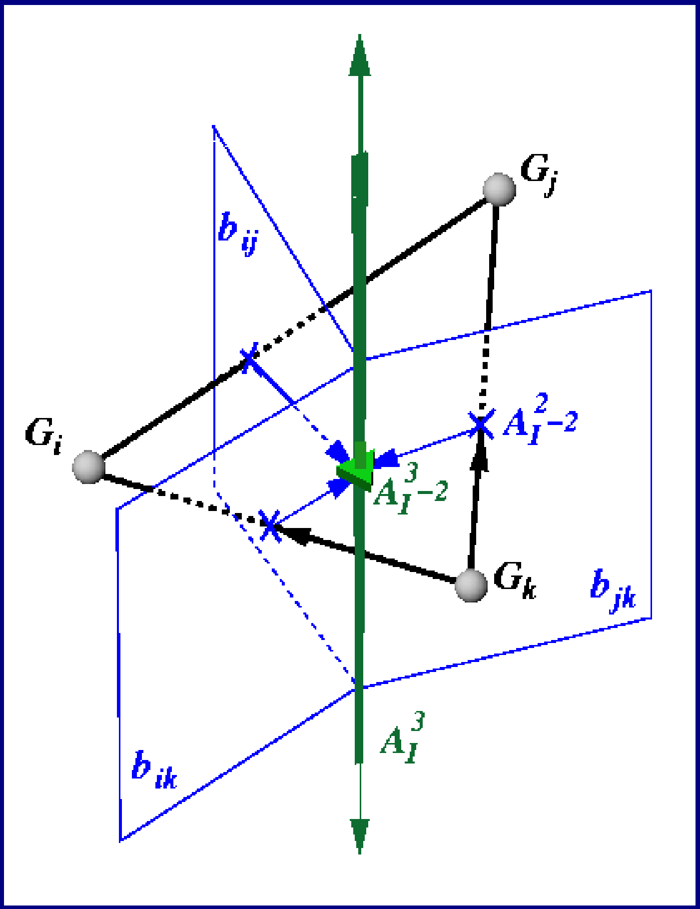


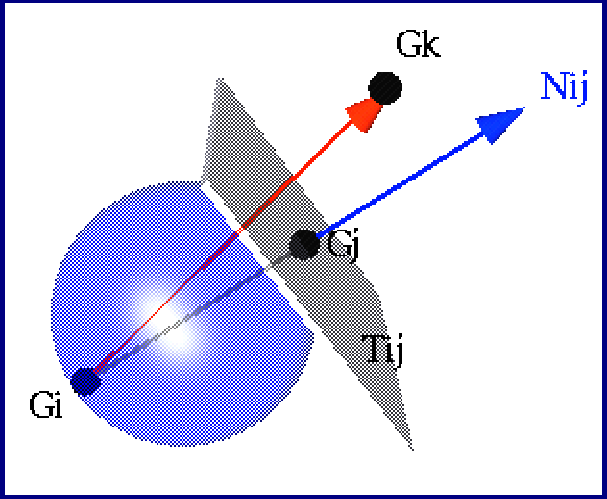
Leymarie-Kima '01: Keep only singular points of the flow (radius) to build a graph.



Computing the Medial Scaffold







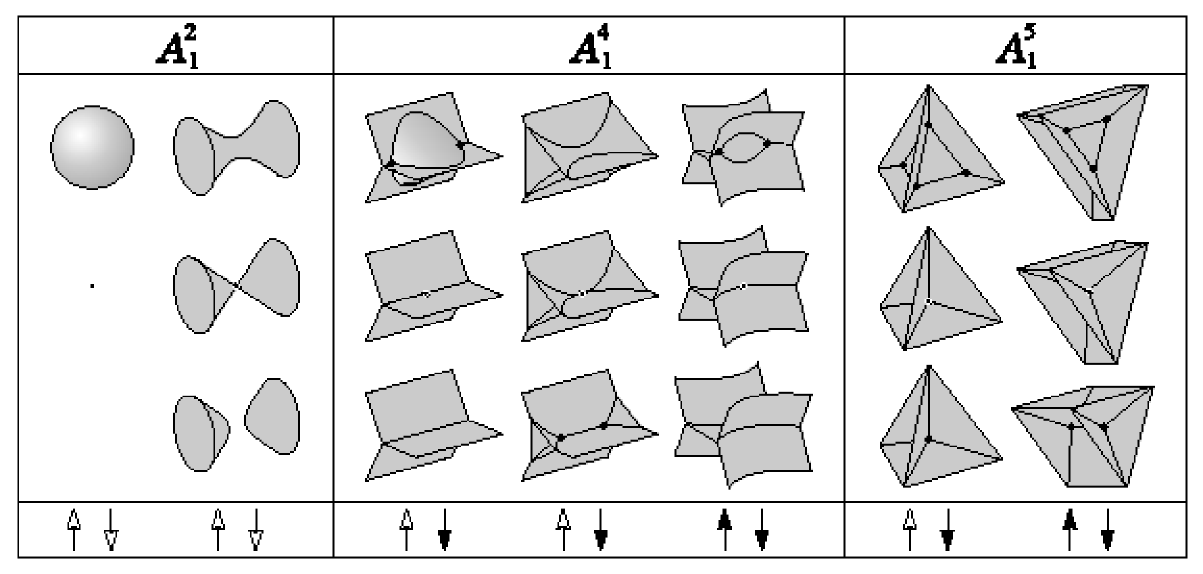
Leymarie-Kimia CVPR03:
Visibility constraints
together with clustering
leads to efficiency in
computing the graph
(linear in practice in the number of input generators)

Transitions of the MA

Study the topological events of the graph structure of the MA under perturbations and shape deformations.

Singularity theory (Arnold, Bogaevsky, since the 1990's):

- In 3D, 26 topologically different perestroikas of linear shock waves.



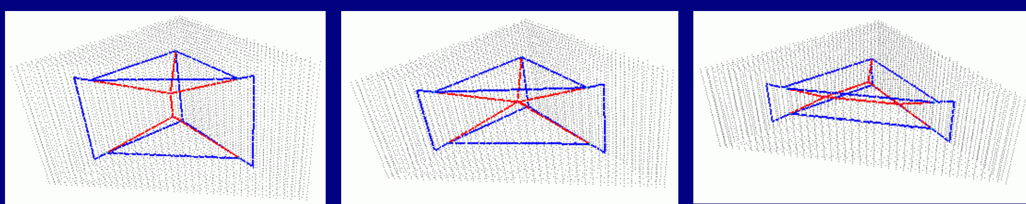
"Perestroikas of shocks and singularities of minimum functions," I. Bogaevsky, 2002.

Transitions of the Medial Axis (MA)

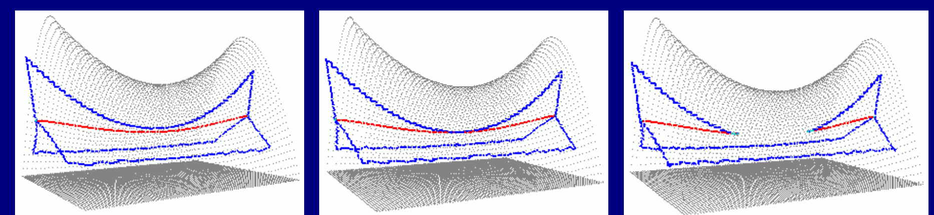
Study the topological events of the graph structure of the MA under perturbations and shape deformations.

Transitions of the MA (Giblin & Kimia, ECCV 2002):

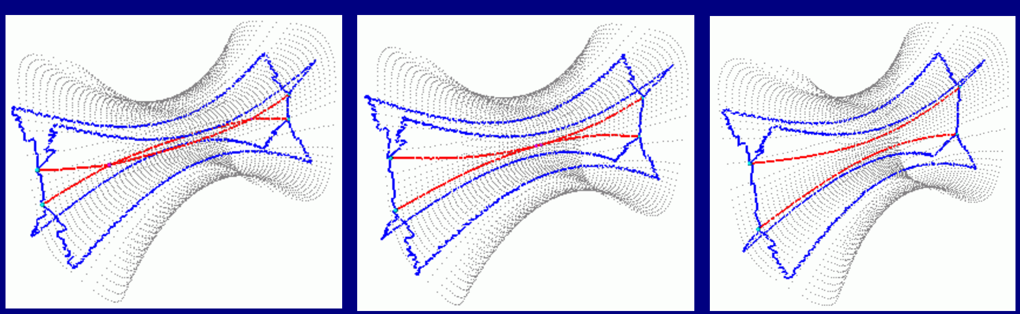
- Under a 1-parameter family of deformations, only **seven transitions** are relevant.



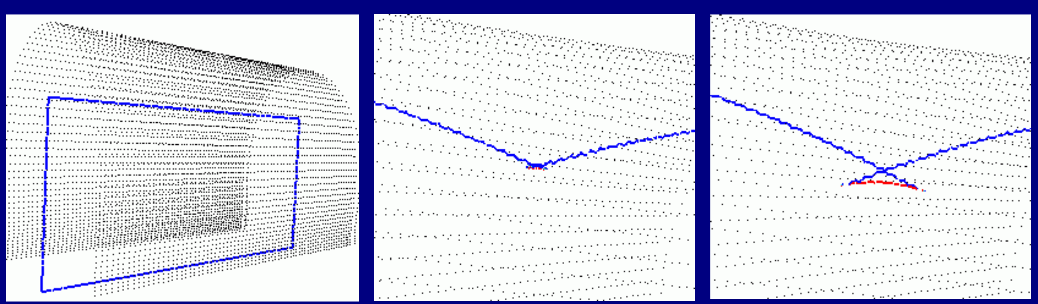
$\mathbf{A_1^5}$ (compression-like)



$\mathbf{A_1 A_3}$**-II** (pulling apart-like)



$\mathbf{A_1^4}$ (compression-like)

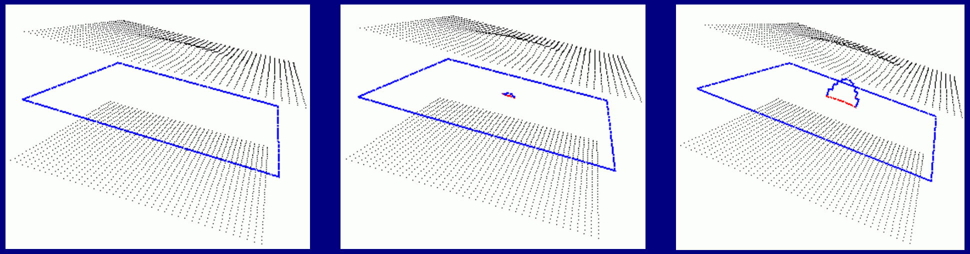


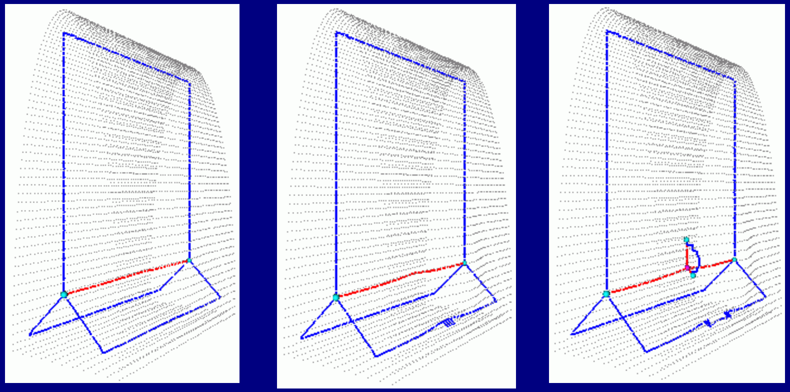
$\mathbf{A_5}$ (ridge merging-like)

Transitions of the Medial Axis (MA)

Study the topological events of the graph structure of the MA under perturbations and shape deformations.

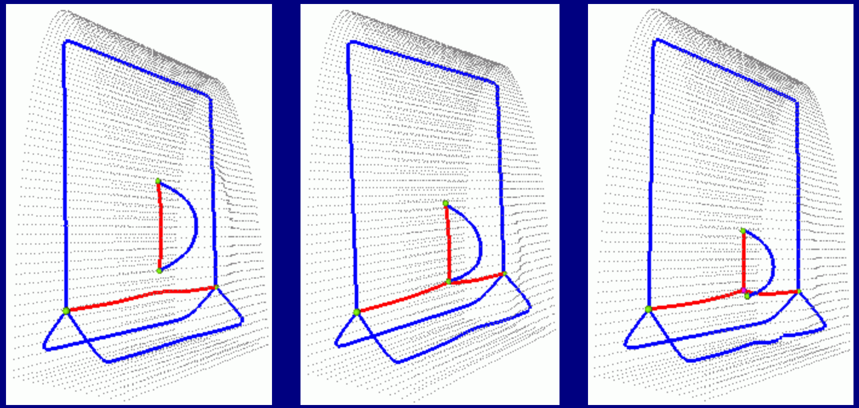
A_1A_3-I (protrusion-like)





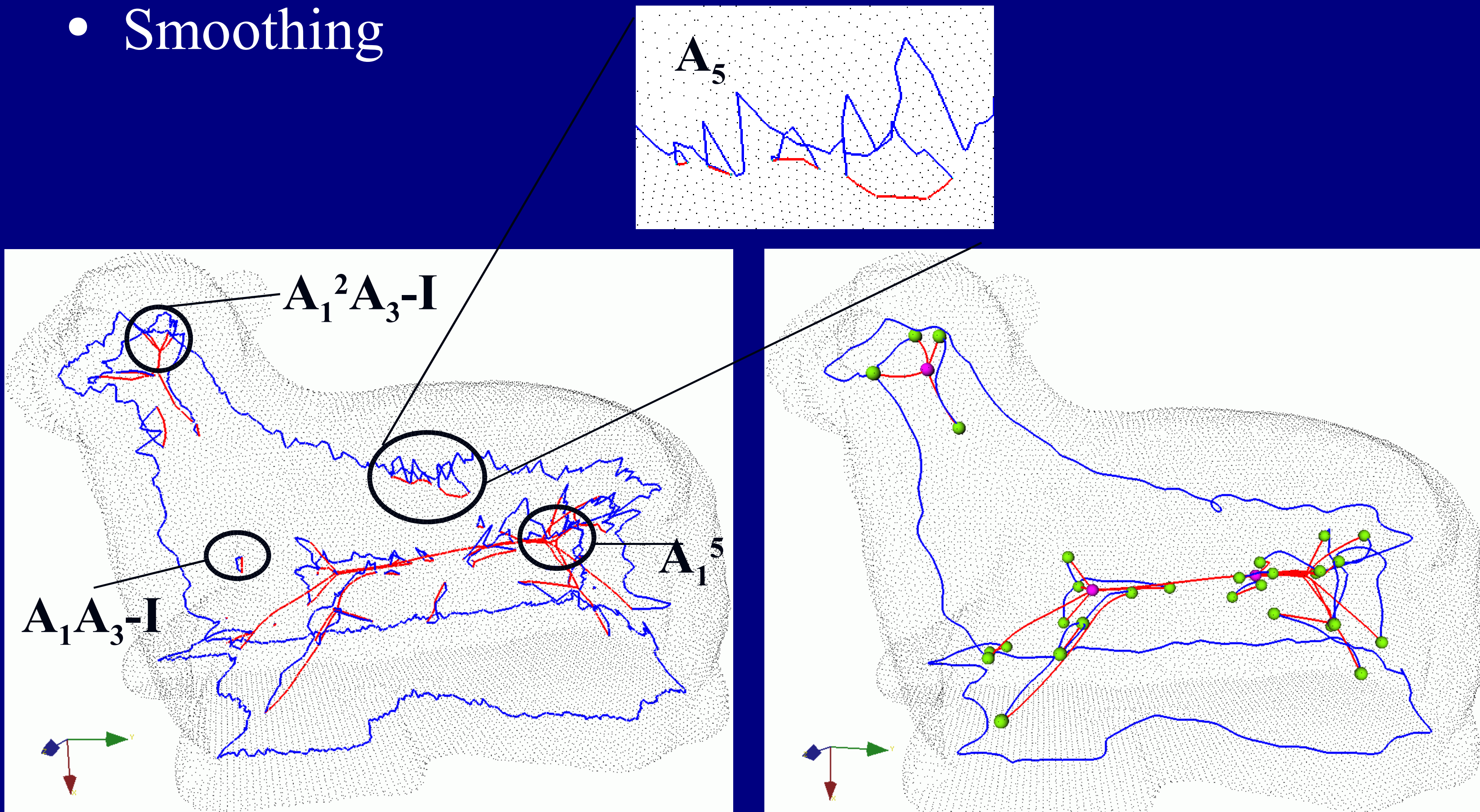
$A_1^2A_3$-I (protrusion on axial curve)

$A_1^2A_3$-II (protrusion moving thru an axial curve)



Scaffold Regularization

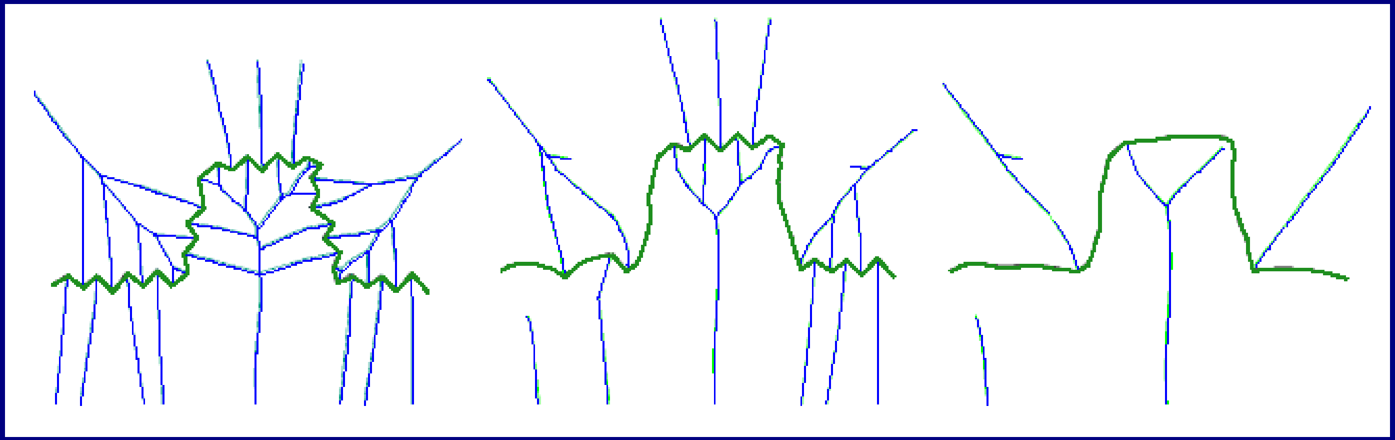
- Transition removal, *i.e.*, remove topological instability
- Smoothing



Blue: A_3 links, **Red**: A_1^3 links   Green: A_1A_3 nodes, Pink: A_1^4 nodes

2D Scaffold Regularization

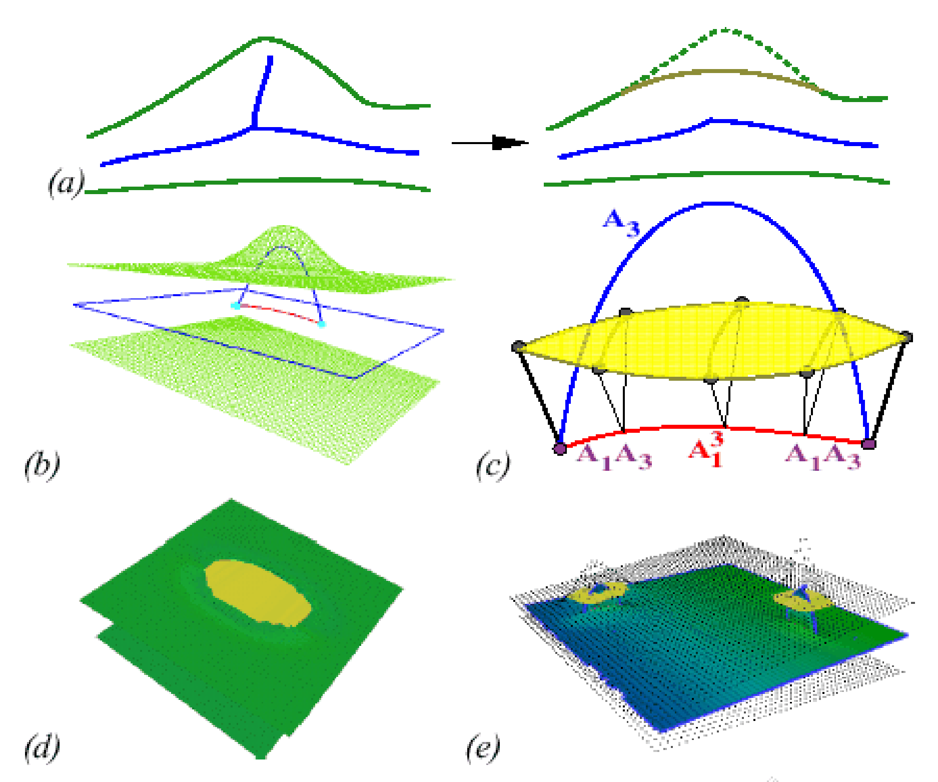
- Transition removal, *i.e.*, remove topological instability
- 2D Boundary Smoothing ordered by "scale" (Tek & Kimia JMIV 2001).



Iterative removal of MA branches, ordered by boundary support (*i.e.*, how much of the contour is represented), coupled with local boundary model adjustment, results in corner enhancement and small perturbations' smoothing.

3D Scaffold Regularization

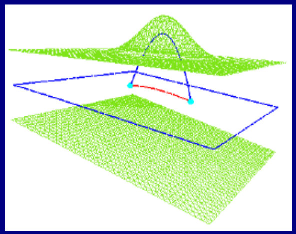
- Transition removal, *i.e.*, remove topological instability
- 3D Boundary Smoothing ordered by "scale."
- Only (3) transitions pertaining to "**protrusions**" are considered here.

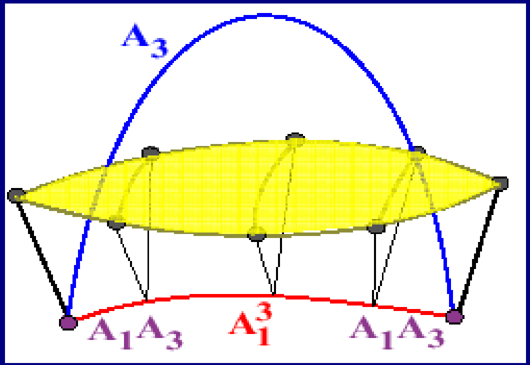


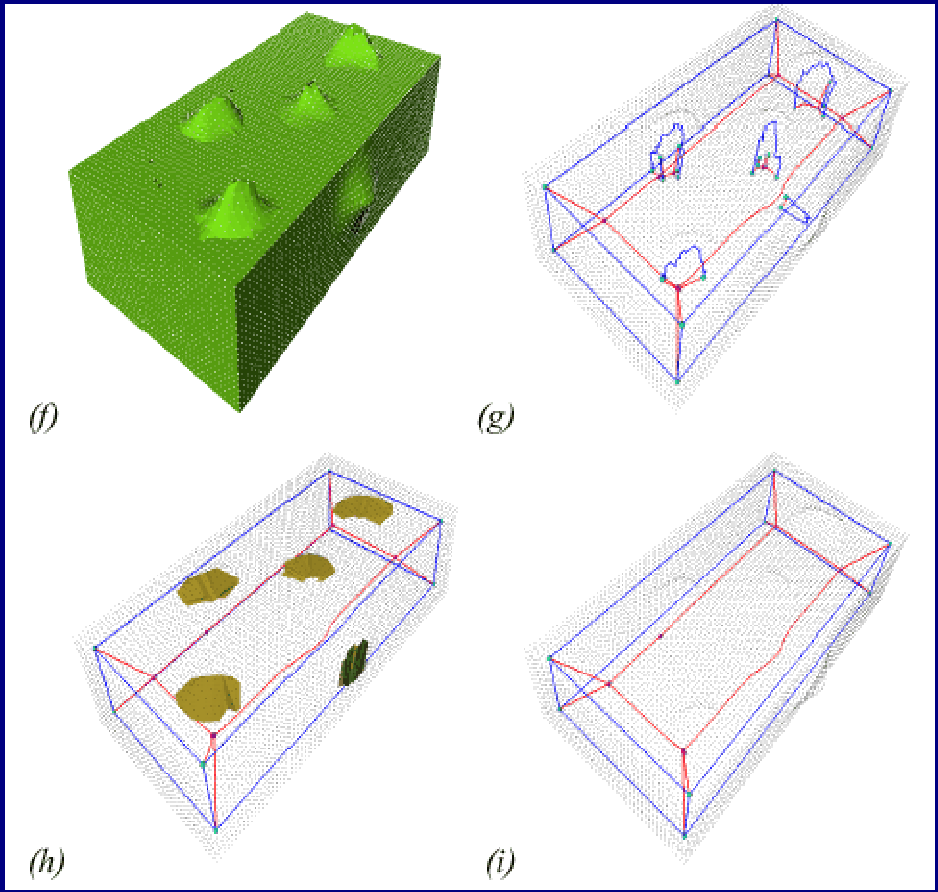
(1) Detect **Axial**-**Rib** loops in the medial scaffold.

(2) Support on the boundary is approximated as the volume subtented by V-shaped hinges which define a cap-like region in between the **axial** and **rib** curves delimiting a potential transition.

3D Scaffold Regularization







(1) **Compute** full Scaffold from unorganized point clouds.

(2) Symmetries pertaining to the interaction of nearby samples are used to mesh the data into **surface** patches (***S***) [Leymarie & Kimia, DIMACS 2003].

(3) Remaining graph structure is the **Medial Scaffold** (MS).

(4) Detect **Axial-Rib loops** in MS.

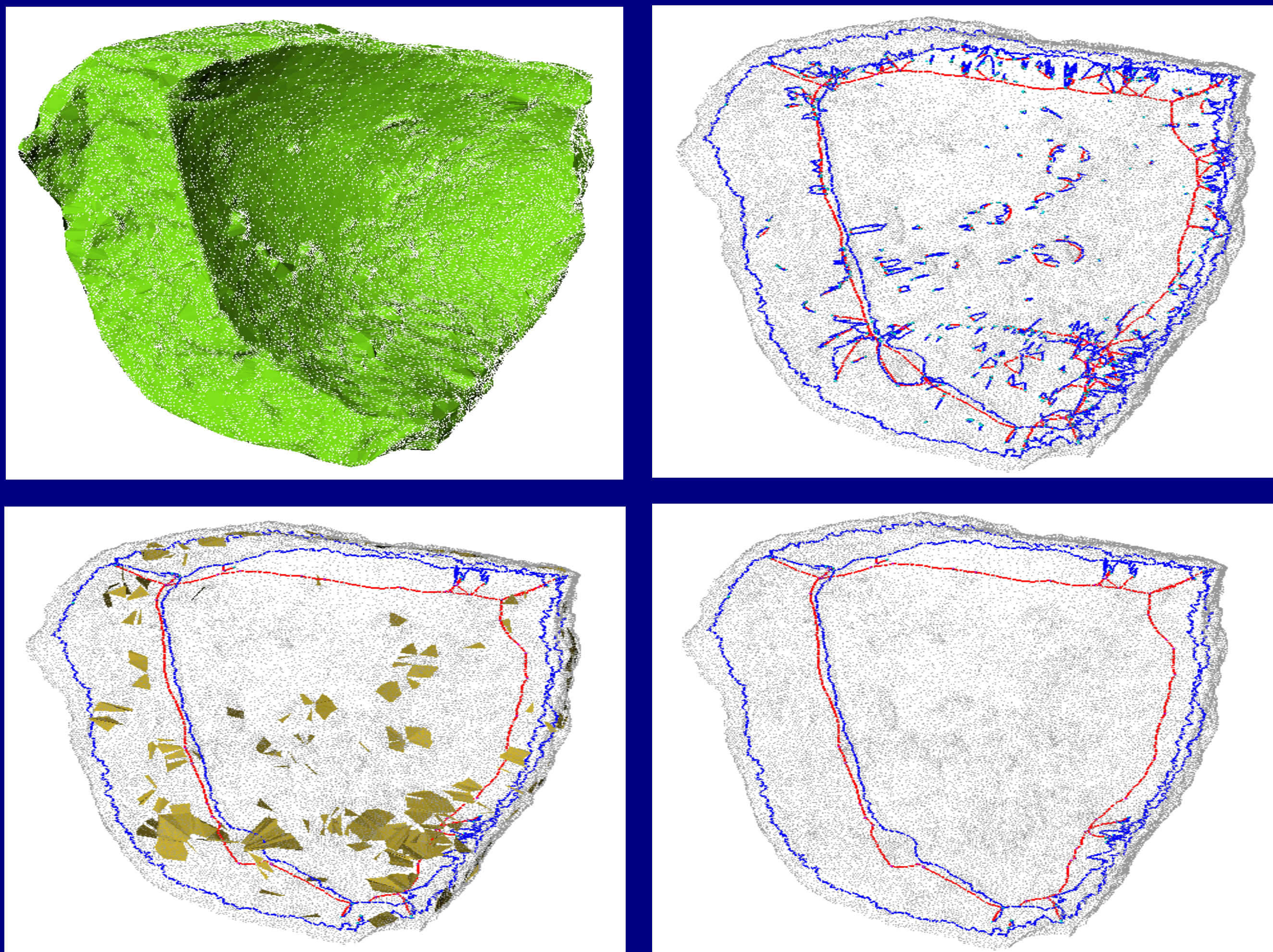
(5) "Walk" along **axial curves** for each loop identifies **contact curves** on ***S***: protrusions limits.

(6) Build smooth **cut-off patches**.

(7) **Rank order** loop structures.

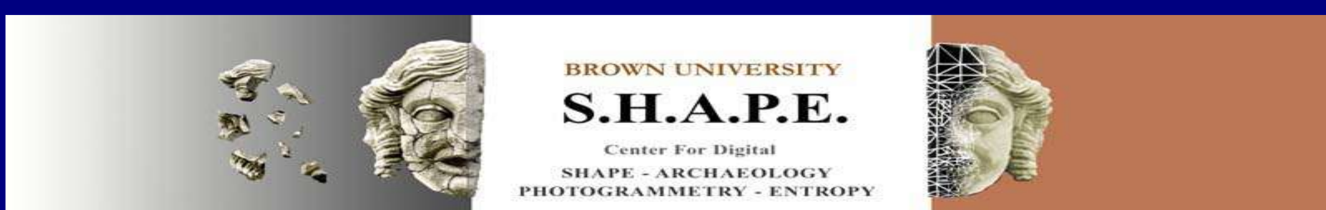
(8) Perform **transition removal**.

3D Scaffold Regularization



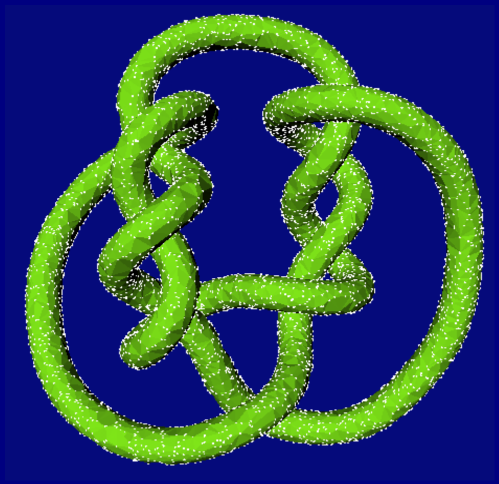
Sherd data: 50K from laser scans

Significant **ridges** useful for pot matching and reconstruction (a 3D puzzle) in digital archaeology applications.

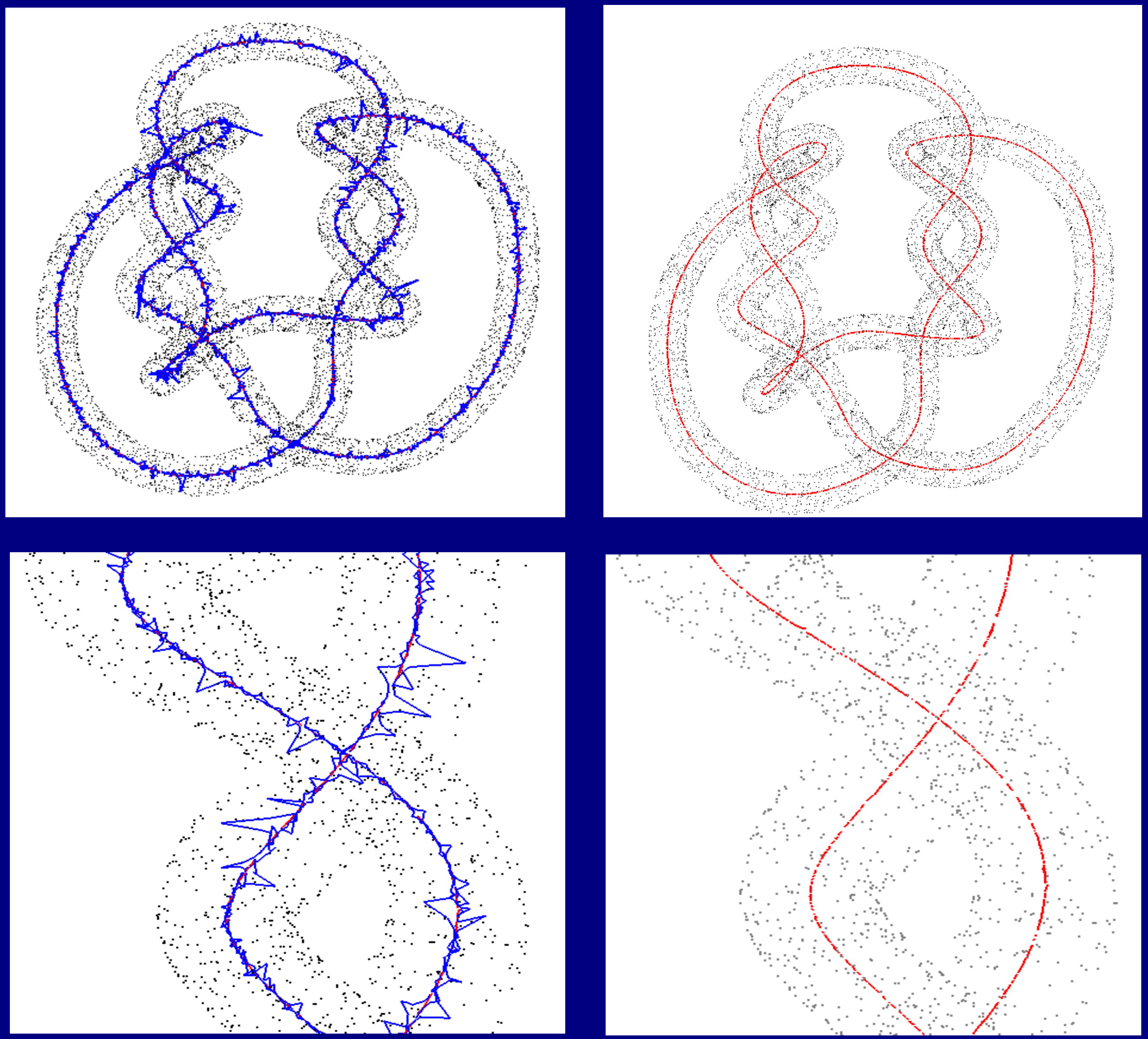


SHAPE Lab. :
www.lems.brown.edu/shape/

3D Scaffold Regularization



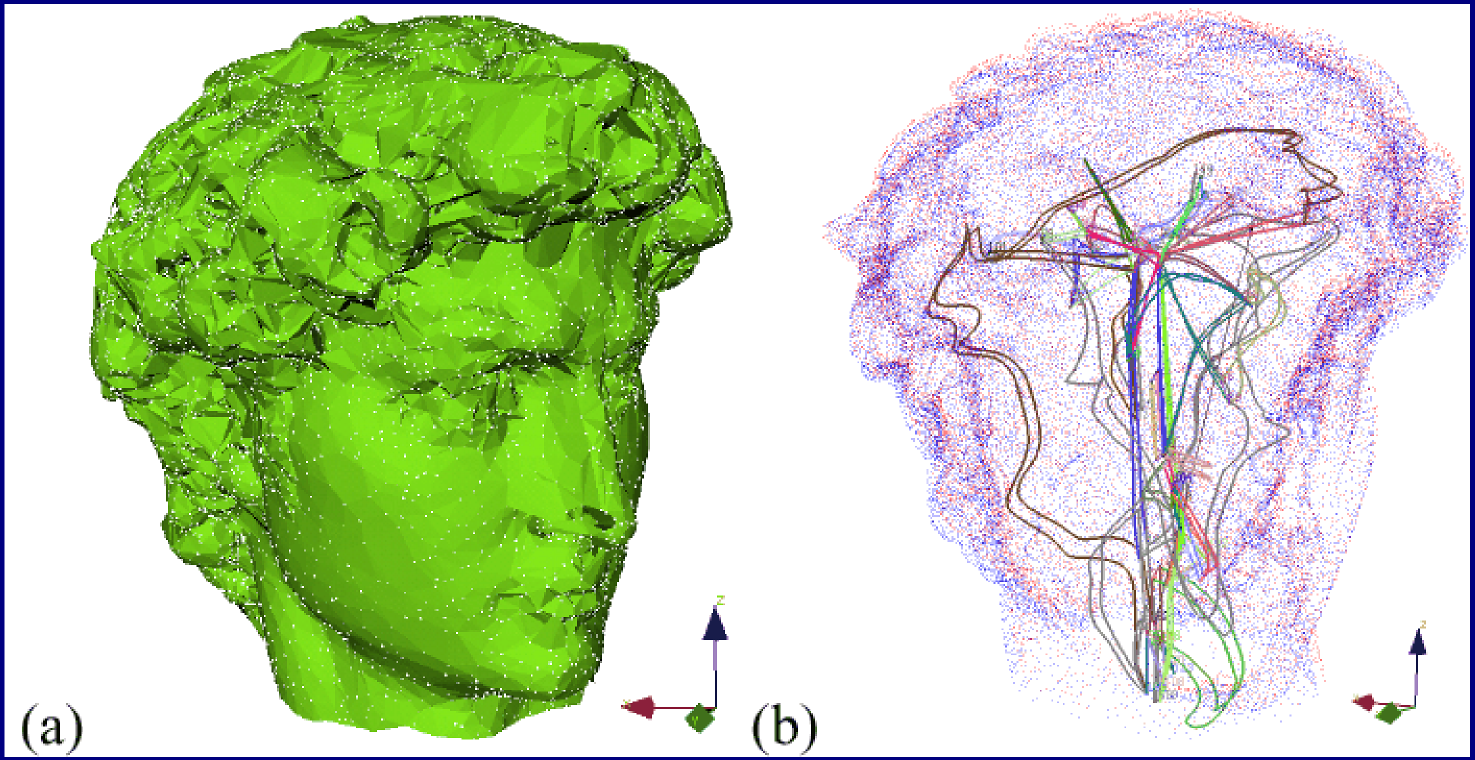
Knot data: 10K random samples



Towards generalized cylinders

Towards 3D Object Recognition

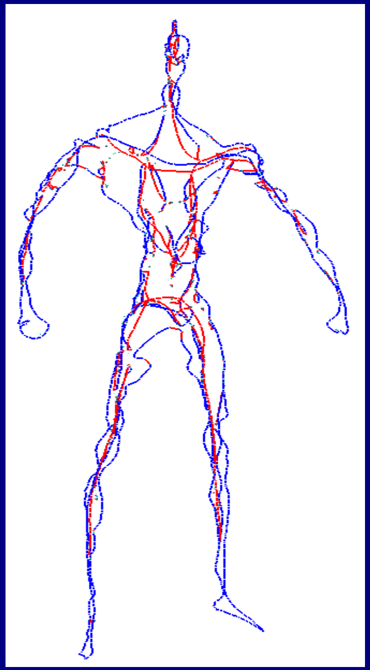
Graph matching via graduated assignement
(to be presented at 3DPVT, Greece, Sept. 2004, Chang, Leymarie & Kimia)
a solution to the **Global Registration** problem.



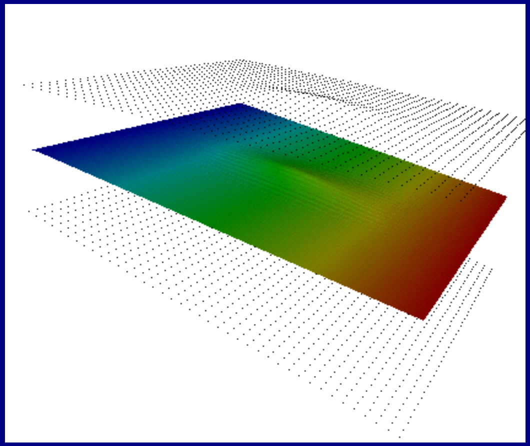
David data: 20K and 30K reduced sample sets (ground truth = 50K)

Validation against ground truth: (object dimension = 69x69x76)
average sq. dist. = 0.000005

Conclusions - Limitations



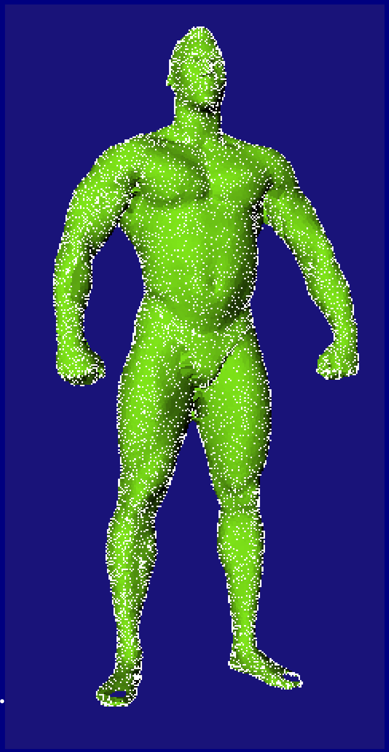
- Perturbations smaller than contact sphere radius are not "visible" from the graph structure alone.



Study curvatures of the MA [Damon, Giblin & Kimia, Siersma].

Study the topography of the MA, *e.g.*, hills & dales [Nackman & Pizer, 1980's]

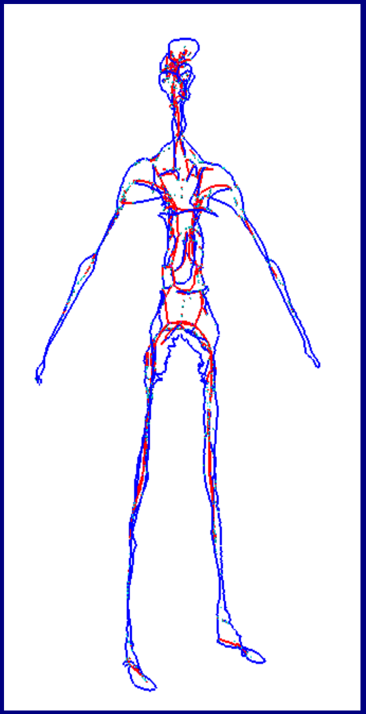
- Complementary to recent anisotropic smoothing methods (explicit ridges).
- Four remaining transitions not yet used for surface regularization.

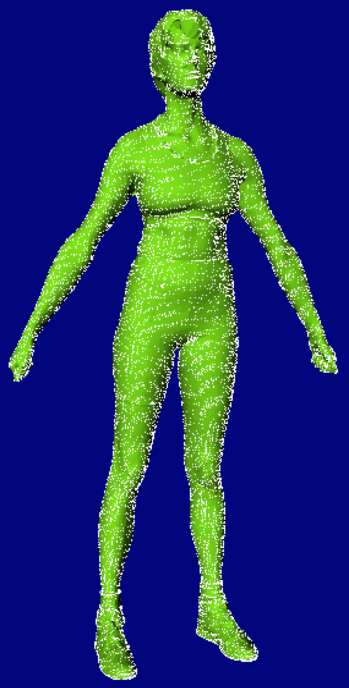


Data from Cyberware Inc.

Conclusions - Features

- Hierarchical representation is built-in.
- Scale represented: to distinguish large features from small ones and rank accordingly.
- Reconstruction of the shape from its skeleton is always possible (exact to approx.).
- Robustness: Structural information from the scaffold is more reliable than surface samples.
- Generalized axes of elongated objects, and ridges are explicit represented.
- Shape deformations are now handled.





Data from
Cyberware Inc.

WESKATMA

What Everybody Should Know About The Medial Axis

- A natural graph structure for the MA exists and leads to efficiency in computation, visualization, applications: the Medial Scaffold hierarchy in 3D.
- The MA is typically NOT overly sensitive to small (scale) perturbations; it does respond to sharpness.
- Shape deformations and perturbations are encoded as topological events, the transitions of shock waves.
- The MA needs not be traced explicitly: the focus should be on the (isolated) singularities of the radius flow.